2.5 The Power Rule

Calculus

Solutions **Practice**

Find $\frac{dy}{dx}$.

1. $y = x^7$

$\frac{dy}{dx} = \boxed{7x^6}$

2. $y = x$

$\frac{dy}{dx} = \boxed{1}$

3. $y = x^\pi$

$\frac{dy}{dx} = \boxed{\pi x^{\pi-1}}$

4. $y = \frac{1}{x^5} = x^{-5}$

$\frac{dy}{dx} = -5x^{-6} = \boxed{-\frac{5}{x^6}}$

5. $y = \frac{1}{\sqrt[4]{x}} = x^{-1/4}$

$\frac{dy}{dx} = -\frac{1}{4}x^{-5/4} = \boxed{-\frac{1}{4\sqrt[4]{x^5}}}$

6. $y = \sqrt[9]{x^4} = x^{4/9}$

$\frac{dy}{dx} = \frac{4}{9}x^{-5/9} = \boxed{\frac{4}{9\sqrt[9]{x^5}}}$

7. $y = \sqrt[3]{x} = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \boxed{\frac{1}{3\sqrt[3]{x^2}}}$

8. $y = x^e$

$\frac{dy}{dx} = \boxed{e x^{e-1}}$

9. $y = \frac{x}{\sqrt[3]{x}} = x^{1-1/3} = x^{2/3}$

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} = \boxed{\frac{2}{3\sqrt[3]{x}}}$

10. $y = x^2\left(\sqrt[6]{x^5}\right) = x^{2+5/6} = x^{17/6}$

$\frac{dy}{dx} = \frac{17}{6}x^{11/6} = \boxed{\frac{17}{6}\sqrt[6]{x^{11}}}$

Find $f'(a)$ for each function at the given value of a.

11. $f(x) = x^4$
find $f'(-1)$

$f'(x) = 4x^3$

$\boxed{f'(-1) = -4}$

12. $f(x) = \sqrt{x}$
find $f'(16)$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\boxed{f'(16) = 1/8}$

13. $f(x) = \frac{1}{x^4}$
find $f'(2)$.

$f'(x) = -4/x^5$

$\boxed{f'(2) = -1/8}$

14. $f(x) = \frac{1}{\sqrt[3]{x}}$
find $f'(27)$.

$f'(x) = -\frac{1}{3(\sqrt[3]{x})^4}$

$\boxed{f'(27) = -\frac{1}{243}}$

Find the equation of the tangent line of each function at the given value of x.

15. $y = x^3$ at $x = -2$

$y' = 3x^2$

$y(-2) = -8$

$y'(-2) = 12$

$\boxed{y + 8 = 12(x+2)}$

16. $f(x) = \sqrt[4]{x^3}$ at $x = 1$

$f'(x) = \frac{3}{4}x^{-\frac{1}{4}} = \frac{3}{4\sqrt[4]{x}}$

$f(1) = 1$

$f'(1) = \frac{3}{4}$

$\boxed{y - 1 = \frac{3}{4}(x-1)}$

17. $f(x) = \frac{1}{x^4}$ at $x = 2$

$f'(x) = -\frac{4}{x^5}$

$f(2) = \frac{1}{16}$

$f'(2) = -\frac{4}{32} = -\frac{1}{8}$

$\boxed{y - \frac{1}{16} = -\frac{1}{8}(x-2)}$

When do the two functions listed have parallel tangent lines?

18. $f(x) = x^2$ and $g(x) = x^5$.

$f'(x) = g'(x)$

$2x = 5x^4$

$0 = 5x^4 - 2x$

$0 = x(5x^3 - 2)$

$\boxed{x = 0,\ x = \sqrt[3]{2/5}}$

19. $f(x) = \sqrt{x}$ and $g(x) = x^3$. Use a calculator.

$f'(x) = g'(x)$

$Y_1 \rightarrow \frac{1}{2\sqrt{x}} = 3x^2 \leftarrow Y_2$

Graph and find intersection.

$\boxed{x = 0.488}$

2.5 The Power Rule

Test Prep

20. $\lim\limits_{x\to e} \frac{(x^3)-(e^3)}{x-e}$ is

Def. of Der. $f'(a) = \lim\limits_{x\to a} \frac{f(x) - f(a)}{x-a}$

$\rightarrow f(x) = x^3$, find $f'(e)$

(A) 0 (B) $3e^2$ (C) e^3 (D) does not exist

21. $\lim\limits_{h\to 0} \frac{\sqrt{(25+h)}-(5)}{h}$ is

Def. of Der. $f'(a) = \lim\limits_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$\rightarrow f(x) = \sqrt{x}$, find $f'(25)$

(A) 0 (B) 5 (C) $\frac{1}{5}$ (D) $\frac{1}{10}$ (E) $\frac{1}{25}$

22. Given $f'(x) = \frac{1}{x}$ and $f(2) = 5$, write an equation for the line which is tangent to the graph of $f(x)$ at the point where $x = 2$.

$f'(2) = \frac{1}{2}$

$y - 5 = \frac{1}{2}(x - 2)$

$y - 5 = \frac{1}{2}x - 1$

(A) $y = \frac{1}{2}x - \frac{1}{2}$ (B) $y = \frac{1}{5}x + 5$ (C) $y = \frac{1}{2}x + 4$

(D) $y = \frac{1}{5}x - \frac{23}{5}$ (E) $y = \frac{1}{2}x + 5$

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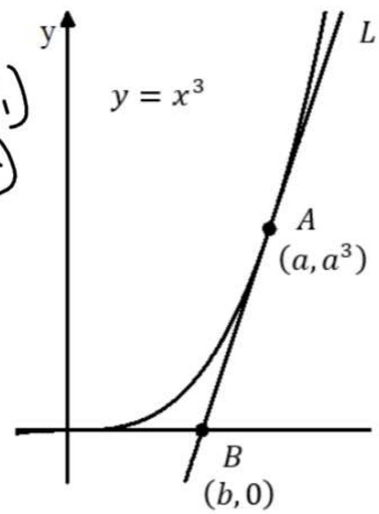
23. In the figure to the right, line L is tangent to the graph of $y = x^3$ at point A with coordinates (a, a^3). Line L crosses the x-axis at point B, with coordinates $(b, 0)$.



a. Find b in terms of a.

$\rightarrow y - y_1 = m(x - x_1)$

$y - a^3 = 3a^2(x - a)$

x-int at $(b, 0)$

$0 - a^3 = 3a^2(b - a)$

$-\frac{a}{3} = b - a$

$\boxed{a - \frac{a}{3} = b}$

b. Find the value of b when $a = 9$.

$9 - \frac{9}{3} = \boxed{6}$