Calculus

2.6 Constant, Const. Multiple, Sum/Difference Notes

Write your questions and thoughts here!

Derivative Rules

Constant: $\frac{d}{dx}c =$

Constant Multiple: $\frac{d}{dx}cu =$

Sum/Difference: $\frac{d}{dx}(u \pm v) =$

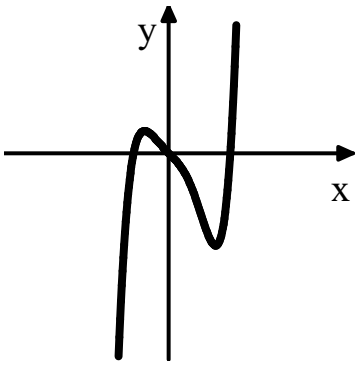
Find the derivative of each function.

1. $y = 2x^2 - \frac{5}{x} + 6$

2. $y = 8\sqrt{x} - \frac{x^6}{3} + 2\pi^5$

Horizontal Tangent Lines

When does a function have a horizontal tangent line?



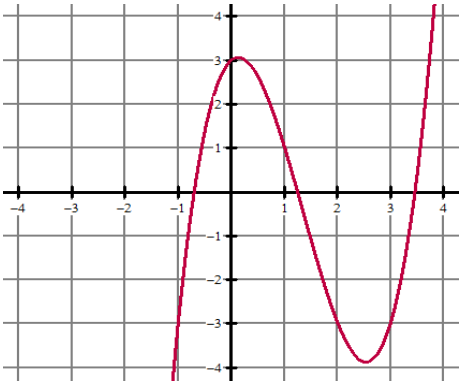
The slope of a horizontal tangent line is zero. To find where a function has a horizontal tangent line, we set the derivative equal to zero.

3. Find the x-values of any horizontal tangent lines of $f(x) = 4x^2 + 7x - 13$.

Normal Lines

A normal line goes through the same point the tangent line does, but it is perpendicular to the tangent line.

4. Find an equation of the NORMAL line of $f(x) = x^3 - 4x^2 + x + 3$ at $x = 3$.



Write your questions and thoughts here!

Differentiability with piecewise functions

5. Is the function $f(x) = \begin{cases} 5x^2 + 3x + 2, & x < -1 \\ -7x - 3, & x \geq -1 \end{cases}$ differentiable at $x = -1$?

6. What values of a and b would make the function $f(x) = \begin{cases} x^2 - ax + 2, & x < 3 \\ x + b, & x \geq 3 \end{cases}$ differentiable at $x = 3$?

2.6 Constant, Constant Multiple, Sum/Difference Rules

Calculus

Practice

Find the derivative of each function.

1. $f(x) = 2x^3 - 4x + 5$	2. $g(x) = 5x^{-2} - \frac{1}{2}x^4$	3. $y = 2e^4 - 3x$	4. $y = \pi x^2 - \pi$
5. $y = 3x^2 - \frac{1}{6x^2}$	6. $h(x) = \frac{x^6}{3} + 6x^{2/3} - 4x^{1/2} + 2$	7. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{5x}$	
8. $f(x) = \sqrt{x} + 3\sqrt[3]{x} + 2$	9. $f(x) = 3x^7 - 4x^3 + 5x + 7$	10. $y = 4\sqrt{x} + e$	

Find the x-value(s) where the function has a horizontal tangent.

11. $f(x) = \frac{x^3}{3} + 4x^2 + 12x - 13$

12. $f(x) = \frac{x^4}{2} + x^3 + \frac{x^2}{2} + 7$

13. $f(x) = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{21}{2}x^2 + \frac{6}{5}$

Find the equations of the tangent AND normal lines of each function at the given value of x.

14. $f(x) = 3\sqrt{x} + 4$ at $x = 4$

Tangent: _____

Normal: _____

15. $y = \frac{x^2}{2} + \frac{3}{2}x - 2$ at $x = 8$

Tangent: _____

Normal: _____

16. $f(x) = -x^3 + 2x^2 - 2$ at $x = 2$

Tangent: _____

Normal: _____

Are the functions differentiable at the given value of x?

17. At $x = 5$.

$$f(x) = \begin{cases} 2x - \frac{8}{5}x^2 + 10, & x \leq 5 \\ 50 - 14x, & x > 5 \end{cases}$$

18. At $x = 9$.

$$f(x) = \begin{cases} \frac{30}{\sqrt{x}} - x^2, & x < 9 \\ x^2 - 5x - 107, & x \geq 9 \end{cases}$$

19. At $x = 3$.

$$f(x) = \begin{cases} 5x^2 - 2x + 1, & x \leq 3 \\ 3x^2 + 2x + 6, & x > 3 \end{cases}$$

What values of a and b would make the function differentiable at the given value of x?

20. At $x = -1$

$$f(x) = \begin{cases} a\sqrt[3]{x} + x^2 - 2, & x < -1 \\ bx + 1, & x \geq -1 \end{cases}$$

21. At $x = 2$.

$$f(x) = \begin{cases} ax^4 + x + 4, & x < 2 \\ bx - 5, & x \geq 2 \end{cases}$$

22. At $x = 1$.

$$f(x) = \begin{cases} \frac{a}{x^2} + x^3 - 2, & x \leq 1 \\ x^2 + bx + 1, & x > 1 \end{cases}$$

2.6 Constant, Constant Multiple, Sum/Difference Rules

Test Prep

23. Given $g(x) = 2x^5 + \frac{b}{x^2}$ where b is a constant, find the value of b if $g'(2) = 180$.

(A) 10 (B) 20 (C) -40 (D) 80 (E) none of these

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24. Calculator required. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

(A) $y = -x - 1.031$
(B) $y = -x - 0.836$
(C) $y = -x + 0.836$
(D) $y = -x + 0.934$
(E) $y = -x + 1.031$

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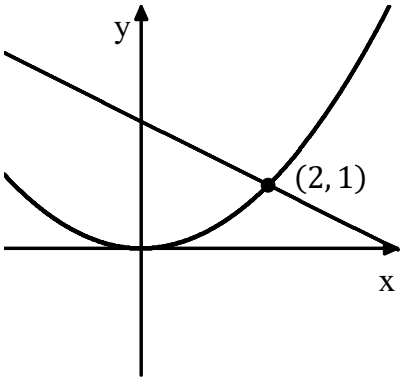
25. $\lim_{h \to 0} \frac{3(x+h)^2 + \frac{2}{x+h} - 3x^2 - \frac{2}{x}}{h}$ is

(A) $x^3 + \frac{2}{x}$ (B) $3x^2 + \frac{2}{x}$ (C) $6x - \frac{2}{x^2}$

(D) $6x + \frac{2}{x^2}$ (E) nonexistent

26. The functions f and g are given by $f(x) = \frac{x^2}{4}$ and $g(x) = -\frac{1}{2}x + 2$.

There is a point P on the graph of f for $x \geq 0$ at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.



27.

$$d(t) = \begin{cases} 20t + t^2 - \frac{t^3}{6}, & 0 \leq t < 3 \\ g(t), & 3 \leq t \leq 16 \end{cases}$$

t (days)	3	8	12	16
$g(t)$ (cubic feet)	64.5	2100	4050	6500

Mr. Bean is building his own swimming pool by digging up his back yard. For the first three days, he uses a shovel. After the 3rd day, he uses a backhoe. The amount of dirt that has been removed, in cubic feet, is modeled by the function d defined above, where g is a differentiable function and t is measured in days. Values of $g(t)$ at selected values of t are given in the table above.

(a) According to the model d, what is the average rate of change of the amount of dirt removed over the time interval $3 \leq t \leq 16$ days?

(b) Use the data in the table to approximate $d'(10)$, the instantaneous rate of change in the amount of dirt removed, in cubic feet per day, at time $t = 10$ days. Show the computations that lead to your answer.

(c) Is d continuous for $0 \leq t \leq 16$? Justify your answer.

(d) Find $d'(2)$. Use appropriate units.