2.6 Constant, Constant Multiple, Sum/Difference Rules

Calculus

Solutions **Practice**

Find the derivative of each function.

1. $f(x) = 2x^3 - 4x + 5$

$$\boxed{f'(x) = 6x^2 - 4}$$

2. $g(x) = 5x^{-2} - \frac{1}{2}x^4$

$$g'(x) = -10x^{-3} - 2x^3$$

$$\boxed{g'(x) = -\frac{10}{x^3} - 2x^3}$$

3. $y = 2e^4 - 3x$

$$\boxed{\frac{dy}{dx} = -3}$$

4. $y = \pi x^2 - \pi$

$$\boxed{\frac{dy}{dx} = 2\pi x}$$

5. $y = 3x^2 - \frac{1}{6x^2}$

$$y = 3x^2 - \frac{1}{6}x^{-2}$$

$$\frac{dy}{dx} = 6x + \frac{2}{6}x^{-3}$$

$$\boxed{\frac{dy}{dx} = 6x + \frac{1}{3x^3}}$$

6. $h(x) = \frac{x^6}{3} + 6x^{2/3} - 4x^{1/2} + 2$

$$h'(x) = \frac{6}{3}x^5 + 6 \cdot \frac{2}{3}x^{-1/3} - 4 \cdot \frac{1}{2}x^{-1/2}$$

$$\boxed{h'(x) = 2x^5 + \frac{4}{\sqrt[3]{x}} - \frac{2}{\sqrt{x}}}$$

7. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{5x}$

$$f(x) = x^{-1/2} + \frac{3}{5}x^{-1}$$

$$f'(x) = -\frac{1}{2}x^{-3/2} - \frac{3}{5}x^{-2}$$

$$\boxed{f'(x) = -\frac{1}{2\sqrt{x^3}} - \frac{3}{5x^2}}$$

8. $f(x) = \sqrt{x} + 3\sqrt[3]{x} + 2$

$$f(x) = x^{1/2} + 3x^{1/3} + 2$$

$$f'(x) = \frac{1}{2}x^{-1/2} + x^{-2/3}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}}$$

9. $f(x) = 3x^7 - 4x^3 + 5x + 7$

$$\boxed{f'(x) = 21x^6 - 12x^2 + 5}$$

10. $y = 4\sqrt{x} + e$

$$\frac{dy}{dx} = 4 \cdot \frac{1}{2}x^{-1/2}$$

$$\boxed{\frac{dy}{dx} = \frac{2}{\sqrt{x}}}$$

Find the x-value(s) where the function has a horizontal tangent.

11. $f(x) = \frac{x^3}{3} + 4x^2 + 12x - 13$

$f'(x) = x^2 + 8x + 12$

$(x+6)(x+2) = 0$

$\boxed{x = -6 \quad x = -2}$

12. $f(x) = \frac{x^4}{2} + x^3 + \frac{x^2}{2} + 7$

$f'(x) = 2x^3 + 3x^2 + x$

$x(2x^2 + 3x + 1) = 0$

$x(2x+1)(x+1) = 0$

$\boxed{x = 0,\ x = -\frac{1}{2},\ x = -1}$

13. $f(x) = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{21}{2}x^2 + \frac{6}{5}$

$f'(x) = x^3 - 10x^2 + 21$

$x(x^2 - 10x + 21) = 0$

$x(x-3)(x-7) = 0$

$\boxed{x = 0,\ x = 3,\ x = 7}$

Find the equations of the tangent AND normal lines of each function at the given value of x.

14. $f(x) = 3\sqrt{x} + 4$ at $x = 4$

$f(4) = 3(2) + 4 = 10$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}}$

$f'(4) = \frac{3}{2} \cdot \frac{1}{\sqrt{4}} = \frac{3}{4}$

Tangent: $y - 10 = \frac{3}{4}(x - 4)$

Normal: $y - 10 = -\frac{4}{3}(x - 4)$

15. $y = \frac{x^2}{2} + \frac{3}{2}x - 2$ at $x = 8$

$y(8) = 32 + 12 - 2 = 42$

$y' = x + \frac{3}{2}$

$y'(8) = 8 + \frac{3}{2} = \frac{19}{2}$

Tangent: $y - 42 = \frac{19}{2}(x - 8)$

Normal: $y - 42 = -\frac{2}{19}(x - 8)$

16. $f(x) = -x^3 + 2x^2 - 2$ at $x = 2$

$f(2) = -8 + 8 - 2 = -2$

$f'(x) = -3x^2 + 4x$

$f'(2) = -12 + 8 = -4$

Tangent: $y + 2 = -4(x - 2)$

Normal: $y + 2 = \frac{1}{4}(x - 2)$

Are the functions differentiable at the given value of x?

17. At $x = 5$.

$$f(x) = \begin{cases} 2x - \frac{8}{5}x^2 + 10, & x \le 5 \\ 50 - 14x, & x > 5 \end{cases}$$

$2(5) - \frac{8}{5}(25) + 10 = 50 - 14(5)$

$10 - 40 + 10 = 50 - 70$

$-20 = -20$ ✓

$2 - \frac{16}{5}x = -14$

$2 - 16 = -14$ ✓

$\boxed{\text{Yes}}$

18. At $x = 9$.

$$f(x) = \begin{cases} \frac{30}{\sqrt{x}} - x^2, & x < 9 \\ x^2 - 5x - 107, & x \ge 9 \end{cases}$$

$\frac{30}{3} - 81 = 81 - 45 - 107$

$-71 = -71$ ✓

$-15x^{-\frac{3}{2}} - 2x = 2x - 5$

$-15(3)^{-3} - 18 = 18 - 5$

$-\frac{5}{9} - 18 = 13$ ✗

$\boxed{\text{No}}$

19. At $x = 3$.

$$f(x) = \begin{cases} 5x^2 - 2x + 1, & x \le 3 \\ 3x^2 + 2x + 6, & x > 3 \end{cases}$$

$5(9) - 6 + 1 = 3(9) + 6 + 6$

$40 = 39$ ✗

Not continuous

$\boxed{\text{No}}$

What values of a and b would make the function differentiable at the given value of x?

20. At $x = -1$

$$f(x) = \begin{cases} a\sqrt[3]{x} + x^2 - 2, & x < -1 \\ bx + 1, & x \geq -1 \end{cases}$$

$-a + 1 - 2 = -b + 1$

$-a = -b + 2$

$a = b - 2$

$\frac{1}{3}ax^{-2/3} + 2x = b$

$\frac{1}{3}a - 2 = b$

$a = 3b + 6$

$b - 2 = 3b + 6$

$-2b = 8$

$\boxed{b = -4 \quad a = -6}$

21. At $x = 2$.

$$f(x) = \begin{cases} ax^4 + x + 4, & x < 2 \\ bx - 5, & x \geq 2 \end{cases}$$

$a(2)^4 + 2 + 4 = 2b - 5$

$16a + 11 = 2b$

$4a(2)^3 + 1 = b$

$32a + 1 = b$

$16a + 11 = 2(32a + 1)$

$9 = 48a$

$\boxed{a = 3/16}$

$b = 32(3/16) + 1$

$\boxed{b = 7}$

22. At $x = 1$.

$$f(x) = \begin{cases} \frac{a}{x^2} + x^3 - 2, & x \leq 1 \\ x^2 + bx + 1, & x > 1 \end{cases}$$

$\frac{a}{1} + 1 - 2 = 1 + b + 1$

$a - 3 = b$

$\frac{-2a}{(1)^3} + 3(1)^2 = 2(1) + b$

$-2a + 1 = b$

$-2a + 1 = a - 3$

$b = 4/3 - 3$

$\boxed{a = 4/3 \quad b = -5/3}$

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Test Prep

-80

23. Given $g(x) = 2x^5 + \frac{b}{x^2}$ where b is a constant, find the value of b if $g'(2) = 180$.

$g'(x) = 10x^4 - \frac{2b}{x^3} \longrightarrow 10(2)^4 - \frac{2b}{2^3} = 180$

$160 - \frac{b}{4} = 180$

$-\frac{b}{4} = 20 \quad b = -80$

(A) 10 (B) 20 (C) −40 (D) 80 (E) none of these

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24. Calculator required. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

(A) $y = -x - 1.031$
(B) $y = -x - 0.836$
(C) $y = -x + 0.836$
(D) $y = -x + 0.934$
(E) $y = -x + 1.031$

$f'(x) = 6x^5 - 4x^3$

$Y_1 = 6x^5 - 4x^3$

$Y_2 = -1$

Find intersection for x-value

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25. $\lim_{h \to 0} \frac{3(x+h)^2 + \frac{2}{x+h} - 3x^2 - \frac{2}{x}}{h}$ is

Def. of Der.

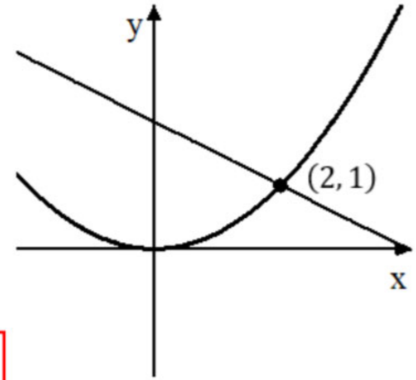
$f(x) = 3x^2 + \frac{2}{x}$

$f'(x) = 6x - 2x^{-2}$

(A) $x^3 + \frac{2}{x}$ (B) $3x^2 + \frac{2}{x}$ (C) $6x - \frac{2}{x^2}$

(D) $6x + \frac{2}{x^2}$ (E) nonexistent

26. The functions f and g are given by $f(x) = \frac{x^2}{4}$ and $g(x) = -\frac{1}{2}x + 2$.



There is a point P on the graph of f for $x \geq 0$ at which the line tangent to the graph of f is $\underbrace{\text{perpendicular}}$ to the graph of g. Find the coordinates of point P.

$\rightarrow$ Slope $= 2$

$f'(x) = 2$

$\frac{2x}{4} = 2$

$x = 4$

$f(4) = 4$

$\boxed{(4,4)}$

27.

$$d(t) = \begin{cases} 20t + t^2 - \frac{t^3}{6}, & 0 \leq t < 3 \\ g(t), & 3 \leq t \leq 16 \end{cases}$$

t (days)	3	8	12	16
$g(t)$ (cubic feet)	64.5	2100	4050	6500

Mr. Bean is building his own swimming pool by digging up his back yard. For the first three days, he uses a shovel. After the 3rd day, he uses a backhoe. The amount of dirt that has been removed, in cubic feet, is modeled by the function d defined above, where g is a differentiable function and t is measured in days. Values of $g(t)$ at selected values of t are given in the table above.

(a) According to the model d, what is the average rate of change of the amount of dirt removed over the time interval $3 \leq t \leq 16$ days?

$$\frac{d(16) - d(3)}{16 - 3} = \frac{6500 - 64.5}{13} = 495.038 \text{ ft}^3/\text{day}$$

(b) Use the data in the table to approximate $d'(10)$, the instantaneous rate of change in the amount of dirt removed, in cubic feet per day, at time $t = 10$ days. Show the computations that lead to your answer.

$$\frac{4050 - 2100}{12 - 8} = \boxed{487.5}$$

(c) Is d continuous for $0 \leq t \leq 16$? Justify your answer.

Yes! $\underset{x \to 3^-}{\lim} \overset{64.5}{d(t)} = \underset{x \to 3^+}{\lim} \overset{64.5}{d(t)} = \overset{64.5}{d(3)}$. Also, $g(t)$ is differentiable, so it must be continuous

(d) Find $d'(2)$. Use appropriate units.

$d'(2) = 20 + 2(2) - \frac{3}{6}(2)^2$

$= 20 + 4 - 2$

$= \boxed{22 \text{ ft}^3/\text{day}}$