2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$

Calculus

Solutions **Practice**

Find the derivative of each function.

1. $f(x) = 2\ln x$

$f'(x) = \frac{2}{x}$

2. $f(x) = 5^x$

$f'(x) = 5^x \ln 5$

3. $f(x) = 9\cos x$

$f'(x) = -9\sin x$

4. $f(x) = 5e^x$

$f'(x) = 5e^x$

5. $f(x) = 8\ln x - 4\cos x + e$

$f'(x) = \frac{8}{x} + 4\sin x$

6. $f(x) = 15\sin x - 3e^x$

$f'(x) = 15\cos x - 3e^x$

7. $f(x) = \log_2 x$

$f'(x) = \frac{1}{x\ln 2}$

8. $f(x) = \log_7 x + \cos x$

$f'(x) = \frac{1}{x\ln 7} - \sin x$

9. $f(x) = 3^x + 3x + x^3$

$f'(x) = 3^x \ln 3 + 3 + 3x^2$

10. $f(x) = 3\sin x$

$f'(x) = 3\cos x$

Find the value of the derivative at the given point.

11. If $f(x) = 3x - 6\cos x$, find $f'\left(\frac{\pi}{2}\right)$

$f'(x) = 3 + 6\sin x$

$f'(\pi/2) = 3 + 6\sin(\pi/2)$

$3 + 6(1) = \boxed{9}$

12. If $f(x) = \sqrt{x} - 2\ln x$, find $f'(4)$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{x}$

$f'(4) = \frac{1}{2\sqrt{4}} - \frac{2}{4}$

$\frac{1}{4} - \frac{2}{4} = \boxed{-\frac{1}{4}}$

13. If $f(x) = 4e^x + 5\sin x$, find $f'(0)$

$f'(x) = 4e^x + 5\cos x$

$f'(0) = 4e^0 + 5\cos(0)$

$= 4 + 5(1) = \boxed{9}$

14. If $f(x) = 2\cos x + e^x$, find $f'(\pi)$

$f'(x) = -2\sin x + e^x$

$f'(\pi) = -2\sin(\pi) + e^\pi$

$\boxed{e^\pi}$

Find the equation of the tangent line at the given x-value.

15. $f(x) = 3\cos x + x$ at $x = \frac{\pi}{2}$

$f(\frac{\pi}{2}) = 3\cos(\frac{\pi}{2}) + \frac{\pi}{2} = \frac{\pi}{2} \leftarrow y_1$

$f'(x) = -3\sin x + 1$

$f'(\frac{\pi}{2}) = -3\sin(\frac{\pi}{2}) + 1 = -2 \leftarrow m$

$\boxed{y - \frac{\pi}{2} = -2(x - \frac{\pi}{2})}$

16. $f(x) = 4e^x - 3\sin x + x^2$ at $x = 0$

$f(0) = 4e^0 - 3\sin(0) + 0^2$
$= 4 - 0 = 4 \leftarrow y_1$

$f'(x) = 4e^x - 3\cos x + 2x$

$f'(0) = 4e^0 - 3\cos(0) + 2(0) = 1 \leftarrow m$

$\boxed{y - 4 = x}$

2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$

Test Prep

17. What is the slope of the line tangent to the graph of $y = 2\ln(x)$ at the point $x = 8$?

$\underbrace{\text{slope of the line tangent}}_{\text{derivative!}}$

$y' = \frac{2}{x} \rightarrow y'(8) = \frac{2}{8}$

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 16 (E) 4

18. If $x = 3$, $\lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h} =$

Def. of Derivative $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

(A) $\ln 3$ (B) 1 (C) $\frac{1}{3}$ (D) nonexistent

19. $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}\right)}{h} =$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f'(\frac{\pi}{6}) = -\sin(\frac{\pi}{6})$

(A) 0 (B) -1 (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{1}{2}$ (E) nonexistent

20. If $f(x) = \sin x$, then $\lim_{x\to\pi} \frac{f(\pi) - f(x)}{x - \pi} =$ ← Careful! The y-values are in a different order than the x-values

$\frac{-[f(x) - f(\pi)]}{x - \pi}$

$f'(x) = \cos x \rightarrow -\cos x$

(A) $-\pi$ (B) -1 (C) 1 (D) π

21. **Calculator active.** Line L is tangent to the graph of $y = 2\sin x$ at the point $(k, 2\sin k)$, where $0 < k < 2\pi$. For what value of k does the line L pass through the origin? $\rightarrow (0,0)$ $\quad y' = 2\cos x$

$y - y_1 = m(x - x_1)$

$y - 2\sin k = 2\cos k(x - k)$

Plug in (0,0)

$0 - 2\sin k = 2\cos k(-k)$ $\rightarrow$ Graph and find point of int.

$Y_1 = -2\sin(k)$

$Y_2 = -2k\cos(k)$

(A) 1.571 (B) 4.493 (C) 4.712 (D) 3.141 (E) There is no such value of k.

$a > 0$ is important!

22. $f(x) = x + a\cos x$, where a is a positive constant and $0 \le x \le 2\pi$. Find the x-coordinates of all points, $0 \le x \le 2\pi$, where the line $y = x + a$ is tangent to the graph of $f(x)$.

Slope of tangent $= f'(x)$

$1 = 1 - a\sin(x)$

$0 = -a\sin(x)$

$a \neq 0$ because $a > 0$.

$\sin x = 0$

$x = 0, \pi, 2\pi$

Tangent line and function must have equivalent y-value

$x + a\cos x = x + a$

$a\cos x = a$

$\cos x = 1$

$x = 0, 2\pi$

$\boxed{x = 0, 2\pi}$