## 2.8 The Product Rule

Calculus

Solutions **Practice**

**Find the derivative of each function.**

1. $f(x) = (2x-3)\sin x$

$\boxed{f'(x) = 2\sin x + (2x-3)\cos x}$

2. $g(x) = 2x^3 e^x$

$\boxed{g'(x) = 6x^2 e^x + 2x^3 e^x}$

3. $h(x) = 4\sqrt{x}\ln x$

$h'(x) = 4\left(\frac{1}{2\sqrt{x}}\right)\ln x + \frac{4\sqrt{x}}{x}$

$h'(x) = \frac{2}{\sqrt{x}}\ln x + \frac{4}{\sqrt{x}}$

$\boxed{h'(x) = \frac{2\ln x + 4}{\sqrt{x}}}$

4. $f(x) = (4-5x)\cos x$

$f'(x) = (-5)\cos x + (4-5x)(-\sin x)$

$\boxed{f'(x) = -5\cos x - (4-5x)\sin x}$

5. $g(x) = 6\ln x \sin x$

$\boxed{g'(x) = \frac{6}{x}\sin x + 6\ln x \cos x}$

6. $h(x) = 2e^x(x^2+x)$

$h'(x) = 2e^x(x^2+x) + 2e^x(2x+1)$

could factor out $2e^x$

$2e^x\left[(x^2+x) + (2x+1)\right]$

$\boxed{h'(x) = 2e^x(x^2+3x+1)}$

7. $f(x) = 8\sin x \cos x$

$f'(x) = 8\cos x\cos x + 8\sin x(-\sin x)$

$\boxed{f'(x) = 8\cos^2 x - 8\sin^2 x}$

8. $g(x) = \frac{3}{x}\ln x$

$g'(x) = -\frac{3}{x^2}\ln x + \frac{3}{x}\left(\frac{1}{x}\right)$

$\boxed{g'(x) = \frac{-3\ln x + 3}{x^2}}$

9. $h(x) = 2x^5\cos x$

$h'(x) = 10x^4\cos x + 2x^5(-\sin x)$

$\boxed{h'(x) = 10x^4\cos x - 2x^5\sin x}$

10. $f(x) = e^x \sin x$

$\boxed{f'(x) = e^x\sin x + e^x\cos x}$

**Use the table to find the value of the derivatives of each function.**

11.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 7 | −5 | 3 | 2 | −3 |

a. $h(t) = f(x)g(x)$
Find $h'(7)$.

$h' = f'g + fg'$

$h'(7) = (3)(2) + (-5)(-3)$

$6 + 15$

$\boxed{21}$

b. $m(x) = 5f(x)g(x)$
Find $m'(7)$.

$m'(x) = 5f'g + 5fg'$

$m'(7) = 5(3)(2) + 5(-5)(-3)$

$30 + 75$

$\boxed{105}$

c. $s(x) = (3f(x)-1)(g(x)+2)$
Find $s'(7)$.

$s'(x) = (3f')(g+2) + (3f-1)(g')$

$s'(7) = (9)(2+2) + (-15-1)(-3)$

$36 + 48$

$\boxed{84}$

12.

| $t$ | $a(t)$ | $a'(t)$ | $b(t)$ | $b'(t)$ |
|---|---|---|---|---|
| −4 | 2 | −3 | −4 | 1 |

a. $f(t) = a(t)b(t)$
Find $f'(-4)$.

$f'(t) = a'b + ab'$

$f'(-4) = (-3)(-4) + (2)(1)$

$12 + 2$

$\boxed{14}$

b. $g(t) = -3a(t)b(t)$
Find $g'(-4)$.

$g'(t) = -3a'b + -3ab'$

$g'(-4) = -3(-3)(-4) - 3(2)(1)$

$-36 - 6$

$\boxed{-42}$

c. $h(t) = (1-a(t))(3b(t)+2)$
Find $h'(-4)$.

$h' = (-a')(3b+2) + (1-a)(3b')$

$h'(-4) = (3)(-12+2) + (1-2)(3)$

$-30 - 3$

$\boxed{-33}$

13.

| $x$ | $d(x)$ | $d'(x)$ | $h(x)$ | $h'(x)$ |
|---|---|---|---|---|
| 1 | $-3$ | $-2$ | 4 | 3 |

a. $a(t) = d(x)h(x)$
Find $a'(1)$.

$a' = d'h + dh'$

$a'(1) = (-2)(4) + (-3)(3)$

$-8 - 9$

$\boxed{-17}$

b. $b(x) = -d(x)h(x)$
Find $b'(1)$.

$b' = -d'h + -dh'$

$b'(1) = (2)(4) - (-3)(3)$

$8 + 9$

$\boxed{17}$

c. $c(x) = \left(2 - \frac{d(x)}{2}\right)(6 - h(x))$
Find $c'(1)$.

$c' = (-\frac{1}{2}d')(6-h) + (2 - \frac{d}{2})(-h')$

$c'(1) = (1)(6-4) + (2 + \frac{3}{2})(-3)$

$2 - \frac{21}{2}$

$\boxed{-\frac{17}{2}}$

**Find the equation of the tangent line at the given $x$-value.**

14. $f(x) = 8 \sin x \cos x$ at $x = \frac{\pi}{3}$

$f(\frac{\pi}{3}) = 8 \sin(\frac{\pi}{3})\cos(\frac{\pi}{3}) = 8\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = 2\sqrt{3}$ $y_1$

$f'(x) = 8\cos x \cos x + 8 \sin x(-\sin x)$

$f'(x) = 8\cos^2 x - 8\sin^2 x$ $m$

$f'(\frac{\pi}{3}) = 8(\frac{1}{2})^2 - 8(\frac{\sqrt{3}}{2})^2 = 8(\frac{1}{4}) - 8(\frac{3}{4}) = -4$

$\boxed{y - 2\sqrt{3} = -4(x - \frac{\pi}{3})}$

15. $g(x) = -2xe^x$ at $x = 0$

$g(0) = -2(0)e^0 = 0 \leftarrow y_1$

$g'(x) = -2e^x + -2xe^x$

$g'(0) = -2e^0 + -2(0)(e^0) = -2 \leftarrow m$

$\boxed{y = -2x}$

## 2.8 The Product Rule

**Test Prep**

16. Let $f$ be a differentiable function with $f(2) = 7$ and $f'(2) = -2$. Let $g$ be the function defined by $g(x) = x^2 f(x)$. Which of the following is an equation of the line tangent to the graph of $g$ at $x = 2$?

$g'(x) = 2xf(x) + x^2 f'(x)$

$g'(2) = 2(2)f(2) + 2^2 f'(2)$

$= 4(7) + 4(-2) = 20 \leftarrow m$

$g(2) = 2^2 f(2)$

$= 4(7)$

$= 28 \leftarrow y_1$

B

(A) $y - 7 = -2(x - 2)$ (B) $y - 28 = 20(x - 2)$ (C) $y = 7(x - 2)$

(D) $y - 7 = 20(x - 2)$ (E) $y - 28 = -2(x - 2)$

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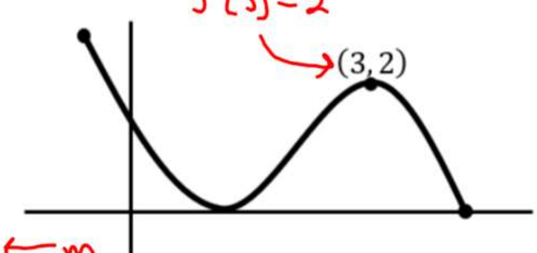
17. The figure to the right shows the graph of $f'$, the derivative of $f$. The function $f$ is twice differentiable with $f(3) = -1$. 17

Let $g$ be the function defined by $g(x) = 4xf(x)$. Find an equation for the line tangent to the graph of $g$ at $x = 3$.

$f'(3) = 2$



Graph of $f'$

$g(3) = 4(3)f(3)$

$= 12(-1)$

$= -12 \leftarrow y_1$

$g' = 4f + 4xf'$

$g'(3) = 4(-1) + 4(3)(2)$

$= -4 + 24 = 20 \leftarrow m$

$\boxed{y + 12 = 20(x - 3)}$

18. The graphs of $f$ and $g$ are shown to the right. If $h(x) = 4f(x)g(x)$, then $h'(1) =$

$g'(x)=2$ $\quad$ $f'(x)=-\frac{1}{2}$

$h'=4f'g+4fg'$

$h'(1)=4f'(1)g(1)+4f(1)g'(1)$

$=4(-\frac{1}{2})(-1)+4(5.5)(2)$

$2+44$

E

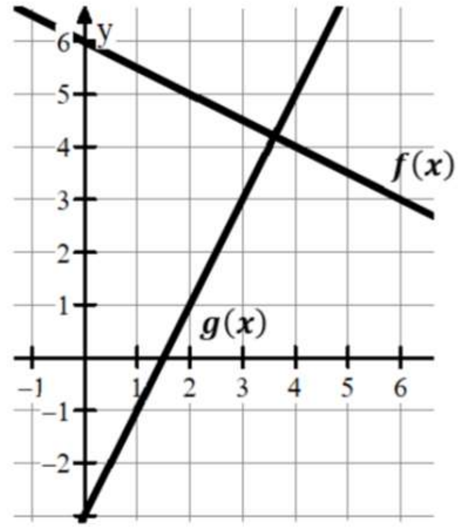
(A) $-22$

(B) $-4$

(C) 0

(D) 4

(E) 46

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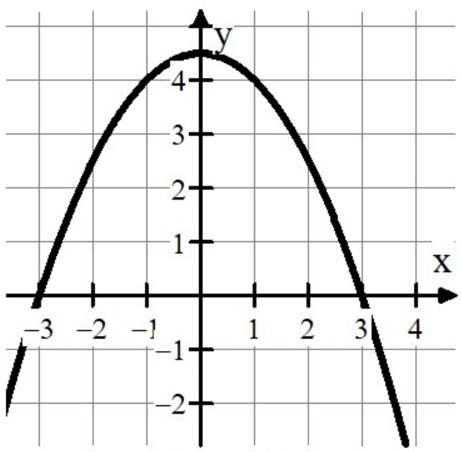
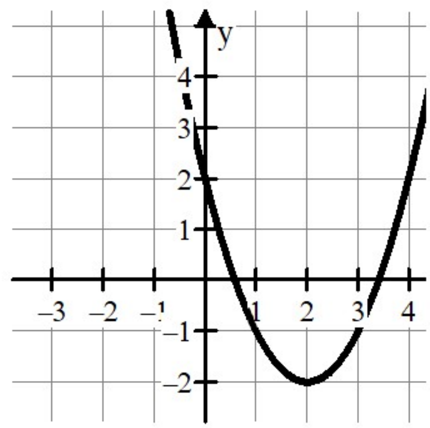
19. The graphs of two differentiable functions $f$ and $g$ are shown below.




**Graph of *f***




**Graph of *g***

Given $h(x) = f(x)g(x)$, which of the following statements about $h'(3)$ is true?

$h'=f'g+fg'$

C

$h'(3)=$ (negative)(negative) + (0)(positive)

$h'(3)=$ positive number

(A) $h'(3) < 0$ $\quad$ (B) $h'(3) = 0$ $\quad$ (C) $h'(3) > 0$

(D) $h'(3)$ is undefined $\quad$ (E) There is not enough information given to conclude anything about $h'(3)$