2.9 The Quotient Rule

Calculus

Solutions

Practice

Find the derivative of each function.

1. $h(x) = \frac{4x-1}{3x+2}$

$$h'(x) = \frac{4(3x+2)-(4x-1)(3)}{(3x+2)^2}$$

$$h'(x) = \frac{12x+8-12x+3}{(3x+2)^2}$$

$$\boxed{h'(x) = \frac{11}{(3x+2)^2}}$$

2. $g(x) = \frac{\sin x}{x}$

$$g'(x) = \frac{\cos x \cdot x - \sin x\,(1)}{x^2}$$

$$\boxed{g'(x) = \frac{x\cos x - \sin x}{x^2}}$$

3. $h(x) = \frac{x^3+2x^2-x}{2x}$

Simplify first!

$$h(x) = \tfrac{1}{2}x^2 + x - \tfrac{1}{2}$$

$$\boxed{h'(x) = x+1}$$

4. $h(x) = \frac{4x}{\ln x}$

$$h'(x) = \frac{4\ln x - 4x\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$\boxed{h'(x) = \frac{4\ln x - 4}{(\ln x)^2}}$$

5. $f(x) = \frac{3x^4-2x^2-3\sqrt{x}}{x}$

Simplify first!

$$f(x) = 3x^3 - 2x - 3x^{-\frac{1}{2}}$$

$$\boxed{f'(x) = 9x^2 - 2 + \tfrac{3}{2}x^{-\frac{3}{2}}}$$

6. $g(x) = \frac{2x^5}{\cos x}$

$$g'(x) = \frac{10x^4\cos x - 2x^5(-\sin x)}{\cos^2 x}$$

$$\boxed{g'(x) = \frac{10x^4\cos x + 2x^5\sin x}{\cos^2 x}}$$

Done, but can you see this answer as well?

$$g'(x) = 10x^4\sec x + 2x^5\tan x\sec x$$

7. $f(x) = \frac{e^x}{4\sin x}$

$$f'(x) = \frac{e^x 4\sin x - e^x 4\cos x}{16\sin^2 x}$$

One way of simplifying:

$$\boxed{f'(x) = \frac{e^x(\sin x - \cos x)}{4\sin^2 x}}$$

8. $f(x) = \frac{2x+4}{3x+2}$

$$f'(x) = \frac{2(3x+2)-(2x+4)(3)}{(3x+2)^2}$$

$$f'(x) = \frac{6x+4-6x-12}{(3x+2)^2}$$

$$\boxed{f'(x) = \frac{-8}{(3x+2)^2}}$$

9. $g(x) = \frac{x^3+3x^2-x}{x^2}$

Simplify first!

$$g(x) = x + 3 - x^{-1}$$

$$\boxed{g'(x) = 1 + \frac{1}{x^2}}$$

Use the table to find the value of the derivatives of each function.

10.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
7	−5	3	2	−3

a. $h(t) = \frac{5f(x)}{g(x)}$ Find $h'(7)$.

$$h'(t) = \frac{5f'g - 5fg'}{g^2}$$

$$h'(7) = \frac{5(3)(2) - 5(-5)(-3)}{4}$$

$$h'(7) = \frac{30-75}{4} = \boxed{\frac{-45}{4}}$$

b. $m(x) = \frac{g(x)+2}{3f(x)}$ Find $m'(7)$.

$$m'(x) = \frac{g'3f - (g+2)3f'}{9f^2}$$

$$m'(7) = \frac{(-3)(3)(-5) - (2+2)3(3)}{9(25)}$$

$$m'(7) = \frac{45-36}{225} = \frac{9}{225} = \boxed{\frac{1}{25}}$$

11.

t	$a(t)$	$a'(t)$	$b(t)$	$b'(t)$
-4	2	-3	-4	1

a. $f(t) = -\frac{b(t)}{3a(t)}$
Find $f'(-4)$.

$f'(t) = -\frac{b'3a - b3a'}{9a^2}$

$f'(-4) = -\frac{(1)3(2) - (-4)3(-3)}{9(4)}$

$f'(-4) = -\frac{6-36}{36} = -\frac{-30}{36} = \boxed{5/6}$

b. $g(t) = \frac{1-a(t)}{2b(t)+3}$
Find $g'(-4)$.

$g' = \frac{-a'(2b+3)-(1-a)(2b')}{(2b+3)^2}$

$g'(-4) = \frac{-(-3)(2(-4)+3)-(1-2)(2\cdot 1)}{(2(-4)+3)^2}$

$g'(-4) = \frac{-15+2}{25} = \boxed{-13/25}$

12.

x	$d(x)$	$d'(x)$	$h(x)$	$h'(x)$
1	-4	-2	4	3

a. $g(x) = \frac{d(x)}{2h(x)}$
Find $g'(1)$.

$g' = \frac{d'2h - d2h'}{4h^2}$

$g'(1) = \frac{(-2)2(\cancel{4}) - (\cancel{-4}^{-1})2(3)}{\cancel{4}(16)}$

$g'(1) = \frac{-4+6}{16} = \frac{2}{16} = \boxed{1/8}$

b. $f(x) = \frac{2-\frac{d(x)}{2}}{6-h(x)}$
Find $f'(1)$.

$f' = \frac{-\frac{1}{2}d'(6-h) - (2-\frac{d}{2})(-h')}{(6-h)^2}$

$f'(1) = \frac{-\frac{1}{2}(-2)(6-4) - (2-\frac{-4}{2})(-3)}{(6-4)^2}$

$f'(1) = \frac{2-(4)(-3)}{4} = \frac{14}{4} = \boxed{7/2}$

Find the equation of the tangent line at the given x-value.

13. $f(x) = \frac{\sin x}{\cos x}$ at $x = \frac{\pi}{3}$

$\boxed{y - y_1 = m(x - x_1)}$

$f(\pi/3) = \frac{\sin \pi/3}{\cos \pi/3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \leftarrow y_1$

$f'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$f'(x) = \frac{1}{\cos^2 x}$ $\quad f'(\pi/3) = \frac{1}{1/4} = 4 \leftarrow m$

$\boxed{y - \sqrt{3} = 4(x - \pi/3)}$

14. $g(x) = -\frac{2x}{e^x}$ at $x = 0$

$g(0) = -\frac{2(0)}{e^0} = 0 \leftarrow y_1$

$g'(x) = -\frac{2e^x - 2xe^x}{e^{2x}}$

$g'(0) = -\frac{2e^0 - 0}{e^0} = -2 \leftarrow m$

$\boxed{y = -2x}$

2.9 The Quotient Rule

Test Prep

15. What is the $\underbrace{\text{instantaneous rate of change}}_{\text{derivative!}}$ at $x = 4$ of the function $f(x) = \frac{x^2-1}{x-2}$?

B

$$f' = \frac{2x(x-2)-(x^2-1)(1)}{(x-2)^2}$$

$$f'(4) = \frac{8(2)-(15)}{4}$$

(A) $-\frac{15}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{15}{2}$

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16. Let f and g be differentiable functions with the following properties:

I. $f(x) < 0$ for all x

II. $g(5) = 2$

If $h(x) = \frac{f(x)}{g(x)}$ and $h'(x) = \frac{f'(x)}{g(x)}$, then $g(x) =$

equivalent

$$h' = \frac{f'g - fg'}{g^2} \text{ must equal } \frac{f'}{g}$$

therefore, f or $g' = 0$.

$f < 0$, so $g' = 0$.

If $g' = 0$, then $g(x) =$ constant

E

(A) $\frac{1}{f'(x)}$ (B) $f(x)$ (C) $-f(x)$ (D) 0 (E) 2

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17. The function f is defined by $f(x) = \frac{x}{x+4}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has a slope of $\frac{1}{9}$?

$$f'(x) = \frac{1(x+4)-x(1)}{(x+4)^2}$$

$$\frac{4}{(x+4)^2} = \frac{1}{9}$$

$36 = (x+4)^2$

$\pm 6 = x + 4$

$x = -10$ or $x = 2$

C

(A) $\left(2, \frac{1}{3}\right)$ only (B) $\left(\frac{1}{9}, \frac{1}{13}\right)$ only (C) $\left(2, \frac{1}{3}\right)$ and $\left(-10, \frac{5}{3}\right)$

$f(2) = 1/3$ $f(-10) = 5/3$

(D) $\left(2, \frac{1}{3}\right)$ and $(-2, -1)$ (E) There are no such points.

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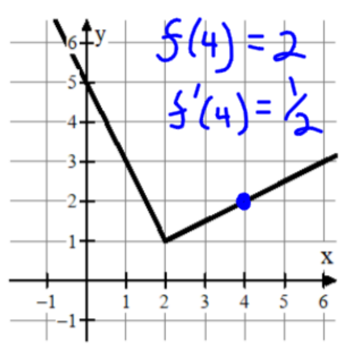
18. The graph of a function f is shown to the right. Let $g(x) = \frac{x^2-1}{f(x)}$. What is the value of $g'(4)$?



Graph of *f*

$$g' = \frac{2xf - (x^2-1)f'}{f^2}$$

$$g'(4) = \frac{2(4)(2)-(16-1)(\frac{1}{2})}{2^2} = \frac{16 - 15/2}{4} = 4 - 15/8$$

$$\frac{32}{8} - \frac{15}{8} = \boxed{17/8}$$