3.1 The Chain Rule

Composite Functions:

$$f(g(x))$$

$$\sin(x^2)$$ 　　　　 $$\sqrt{\ln x}$$ 　　　　 $$\cos(\sin(5x))$$

The Chain Rule (derivative of a composite function)

$$\frac{d}{dx}f(g(x)) =$$

Find the derivative

1. $f(x) = (x^2 - 5)^4$	2. $g(x) = \sqrt{4x - 3}$	3. $h(x) = \sin^2 5x$

4. $y = \ln(x^3)$

5. $y = \ln(x^3)$

6. $f(x) = \left(\frac{t^2+1}{2t-5}\right)^3$

7. If $g(x) = 2x\sqrt{1-x}$ find $g'(-3)$.

8. Given the following table of values, find $f'(4)$ for each function.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
3	−1	7	−2	−3
4	3	−2	9	5

$$f(x) = (g(x))^2 \qquad\qquad f(x) = \sqrt{h(x)} \qquad\qquad f(x) = h(g(x))$$

3.1 The Chain Rule
Calculus

Practice

Find the derivative of each function.

1. $g(x) = (3x^2 - 1)^5$

2. $y = \sin 2x$

3. $h(r) = \sqrt[3]{5r^2 - 2r + 1}$

4. $y = \sqrt{4 - \cos(x^2)}$

5. $h(x) = \ln(5^x)$

6. $g(x) = \ln(2x^3)$

7. $f(x) = \sqrt{\tan(2x)}$

8. $y = \cos^2 x$

9. $y = \dfrac{1}{(7x^2 - 1)^2}$

10. $f(x) = 3^{\sqrt{x}}$

11. $y = \sin^3(4x)$

12. $y = e^{\sqrt{1-\cos x}}$

13. $g(x) = e^{\cos(7x^3)}$

14. $h(x) = \sin(\ln(x^5))$

Find $f'(5)$ given the following.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
5	9	6	5	−4
9	2	−3	−4	1

15. $f(x) = h\big(g(x)\big)$

16. $f(x) = \big(h(x)\big)^2$

17. $f(x) = \sqrt{g(x)}$

18. $f(x) = 2g(x)h(x)$

19. $f(x) = \dfrac{1}{h(x)}$

20. $f(x) = g\big(h(x)\big)$

Find the slope of the tangent line at the given x-value. Show work.

21. $h(x) = \frac{(3x-4)^2}{x}$ at $x = -2$.

22. $g(x) = \cos(\tan x)$ at $x = \pi$.

23. $f(x) = \sqrt{1 + (x^2 - 1)^3}$ at $x = 2$.

Find the equation of the tangent line at the given x-value.

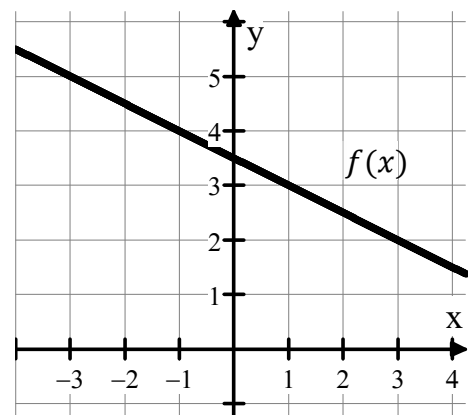24. $f(x) = \sqrt{x^2 - 9}$ at $x = 5$.

25. $g(x) = e^{x^2}$ at $x = 1$.

26. $y = \sin^2(3x)$ at $x = \frac{\pi}{4}$.

3.1 The Chain Rule

Test Prep

27. The graph of the function f is shown at the right.

The function h is defined by $h(x) = f(2x^2 - x)$. Find the slope of the line tangent to the graph of h at the point where $x = -1$

28. Let $f(x) = 2e^{3x}$ and $g(x) = 5x^3$. At what value of x do the graphs of f and g have parallel tangents?

(A) -0.445 (B) -0.366 (C) -0.344 (D) -0.251 (E) -0.165

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29. Let f be the function given by $f(x) = 5e^{3x^3}$. For what positive value of a is the slope of the line tangent to the graph of f at $(a, f(a))$ equal to 6?

(A) 0.142   (B) 0.344   (C) 0.393   (D) 0.595   (E) 0.714

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30. Let $f(x) = \sqrt{2x}$. If the rate of change of f at $x = c$ is four times its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{16}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) 1   (E) 32

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31. Let $f(x) = x \cdot g(h(x))$ where $g(4) = 2, g'(4) = 3, h(3) = 4,$ and $h'(3) = -2$. Find $f'(3)$.

(A) -18 (B) -16 (C) -7 (D) 7   (E) 11