3.1 The Chain Rule

Calculus

Solutions

Practice

Find the derivative of each function.

1. $g(x) = (3x^2 - 1)^5$

$g'(x) = 5(3x^2-1)^4 \cdot (6x)$

$\boxed{g'(x) = 30x(3x^2-1)^4}$

2. $y = \sin 2x$

$\frac{dy}{dx} = \cos(2x) \cdot 2$

$\boxed{\frac{dy}{dx} = 2\cos(2x)}$

3. $h(r) = \sqrt[3]{5r^2 - 2r + 1}$

$h'(r) = \frac{1}{3}(5r^2-2r+1)^{-2/3} \cdot (10r-2)$

$\boxed{h'(r) = \frac{10r-2}{3\sqrt[3]{(5r^2-2r+1)^2}}}$

4. $y = \sqrt{4 - \cos(x^2)}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-\cos(x^2)}} \cdot (\sin(x^2)) \cdot 2x$

$\boxed{\frac{dy}{dx} = \frac{x\sin(x^2)}{\sqrt{4-\cos(x^2)}}}$

5. $h(x) = \ln(5^x)$

$h'(x) = \frac{1}{5^x} \cdot 5^x \ln 5$

$\boxed{h'(x) = \ln 5}$

6. $g(x) = \ln(2x^3)$

$g'(x) = \frac{1}{2x^3} \cdot 6x^2$

$\boxed{g'(x) = \frac{3}{x}}$

7. $f(x) = \sqrt{\tan(2x)}$

$f'(x) = \frac{1}{2\sqrt{\tan(2x)}} \cdot \sec^2(2x) \cdot 2$

$\boxed{f'(x) = \frac{\sec^2(2x)}{\sqrt{\tan(2x)}}}$

8. $y = \cos^2 x$

$\frac{dy}{dx} = 2\cos x(-\sin x)$

$\boxed{\frac{dy}{dx} = -2\cos x \sin x}$

9. $y = \frac{1}{(7x^2-1)^2} = (7x^2-1)^{-2}$

$\frac{dy}{dx} = -2(7x^2-1)^{-3} \cdot (14x)$

$\boxed{\frac{dy}{dx} = \frac{-28x}{(7x^2-1)^3}}$

10. $f(x) = 3^{\sqrt{x}}$

$f'(x) = 3^{\sqrt{x}} \ln 3 \cdot \frac{1}{2\sqrt{x}}$

$$\boxed{f'(x) = \frac{3^{\sqrt{x}} \ln 3}{2\sqrt{x}}}$$

11. $y = \sin^3(4x)$

$\frac{dy}{dx} = 3\sin^2(4x) \cdot \cos(4x) \cdot 4$

$$\boxed{\frac{dy}{dx} = 12\sin^2(4x)\cos(4x)}$$

12. $y = e^{\sqrt{1-\cos x}}$

$\frac{dy}{dx} = e^{\sqrt{1-\cos x}} \cdot \frac{1}{2\sqrt{1-\cos x}} \cdot \sin x$

$$\boxed{\frac{dy}{dx} = \frac{\sin x \, e^{\sqrt{1-\cos x}}}{2\sqrt{1-\cos x}}}$$

13. $g(x) = e^{\cos(7x^3)}$

$g'(x) = e^{\cos(7x^3)} \cdot (-\sin(7x^3)) \cdot (21x^2)$

$$\boxed{g'(x) = -21x^2 \sin(7x^3) e^{\cos(7x^3)}}$$

14. $h(x) = \sin(\ln(x^5))$

$h'(x) = \cos(\ln(x^5)) \cdot \frac{1}{x^5} \cdot 5x^4$

$$\boxed{h'(x) = \frac{5}{x} \cos(\ln(x^5))}$$

Find $f'(5)$ given the following.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
5	9	6	5	−4
9	2	−3	−4	1

15. $f(x) = h(g(x))$

$f' = h'(g(x)) \cdot g'(x)$

$f'(5) = h'(9) \cdot 6$

$(1) \cdot 6 = \boxed{6}$

16. $f(x) = (h(x))^2$

$f'(x) = 2h(x) \cdot h'(x)$

$f'(5) = 2(5) \cdot (-4)$

$\boxed{-40}$

17. $f(x) = \sqrt{g(x)}$

$f'(x) = \frac{1}{2\sqrt{g(x)}} \cdot g'(x)$

$f'(5) = \frac{1}{2\sqrt{9}} \cdot 6$

$\frac{1}{6} \cdot 6 = \boxed{1}$

18. $f(x) = 2g(x)h(x)$

$f'(x) = 2g'(x)h(x) + 2g(x)h'(x)$

$f'(5) = 2(6)(5) + 2(9)(-4)$

$60 - 72$

$\boxed{-12}$

19. $f(x) = \frac{1}{h(x)} = [h(x)]^{-1}$

$f' = -\frac{1}{[h(x)]^2} \cdot h'(x)$

$f' = -\frac{1}{25} \cdot (-4) = \boxed{\frac{4}{25}}$

20. $f(x) = g(h(x))$

$f' = g'(h(x)) \cdot h'(x)$

$f'(5) = g'(5) \cdot (-4)$

$6 \cdot -4$

$\boxed{-24}$

Find the slope of the tangent line at the given x-value. Show work.

21. $h(x) = \frac{(3x-4)^2}{x}$ at $x = -2$.

$h' = \frac{2(3x-4)(3)x - (3x-4)^2}{x^2}$

$h'(-2) = \frac{2(-10)(-6) - (-10)^2}{4}$

$= \frac{120 - 100}{4}$

$\boxed{5}$

22. $g(x) = \cos(\tan x)$ at $x = \pi$.

$g' = -\sin(\tan x) \cdot \sec^2 x$

$g'(\pi) = -\sin(\tan \pi) \cdot \frac{1}{\cos^2(\pi)}$

$= -\sin(0) \cdot \frac{1}{(-1)^2}$

$= -0 \cdot 1$

$\boxed{0}$

23. $f(x) = \sqrt{1 + (x^2 - 1)^3}$ at $x = 2$.

$f' = \frac{3(x^2-1)^2(2x)}{2\sqrt{1+(x^2-1)^3}}$

$f'(2) = \frac{3(3)^2(4)}{2\sqrt{1+(3)^3}}$

$\frac{54}{\sqrt{28}}$

$\boxed{\frac{27}{\sqrt{7}}}$

Find the equation of the tangent line at the given x-value.

24. $f(x) = \sqrt{x^2 - 9}$ at $x = 5$.

$f(5) = \sqrt{16} = 4$

$f'(x) = \frac{1}{\cancel{2}\sqrt{x^2-9}} \cdot \cancel{2}x$

$f'(5) = \frac{5}{4}$

$\boxed{y - 4 = \frac{5}{4}(x - 5)}$

25. $g(x) = e^{x^2}$ at $x = 1$.

$g(1) = e$

$g'(x) = 2x\, e^{x^2}$

$g'(1) = 2e$

$\boxed{y - e = 2e(x-1)}$

26. $y = \sin^2(3x)$ at $x = \pi/4$.

$y\left(\frac{\pi}{4}\right) = \sin^2\left(\frac{3\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

$y' = 2\sin(3x) \cdot \cos(3x) \cdot 3$

$y'\left(\frac{\pi}{4}\right) = 6\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right)$

$6\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

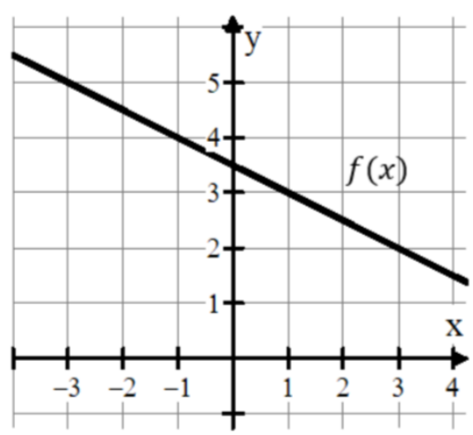
-3

$\boxed{y - \frac{1}{2} = -3\left(x - \frac{\pi}{4}\right)}$

3.1 The Chain Rule

Test Prep

27. The graph of the function f is shown at the right.



The function h is defined by $h(x) = f(2x^2 - x)$. Find the slope of the line tangent to the graph of h at the point where $x = -1$

$h(-1) = f(2(-1)^2 + 1) = f(3) = 2$

$h'(x) = f'(2x^2 - x) \cdot (4x - 1)$

$h'(-1) = f'(3) \cdot (-5)$

$(-\frac{1}{2}) \cdot (-5) = \frac{5}{2}$

$\boxed{y - 2 = \frac{5}{2}(x+1)}$

28. Let $f(x) = 2e^{3x}$ and $g(x) = 5x^3$. At what value of x do the graphs of f and g have parallel tangents?

means derivatives are equal (same slope)

$f'(x) = 6e^{3x}$ $\quad$ $g'(x) = 15x^2$

graph and find pt. of intersection

B

(A) -0.445 (B) -0.366 (C) -0.344 (D) -0.251 (E) -0.165

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29. Let f be the function given by $f(x) = 5e^{3x^3}$. For what positive value of a is the slope of the line tangent to the graph of f at $(a, f(a))$ equal to 6? $\rightarrow f'(a) = 6$

B

Graph and find pt. of intersection

$f'(x) = 45x^2e^{3x^3}$

$Y_1 \rightarrow 45a^2e^{3a^3} = 6 \leftarrow Y_2$

(A) 0.142 (B) 0.344 (C) 0.393 (D) 0.595 (E) 0.714

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30. Let $f(x) = \sqrt{2x}$. If the rate of change of f at $x = c$ is four times its rate of change at $x = 1$, then $c =$

A

$f'(x) = \frac{1}{2\sqrt{2x}} \cdot 2 = \frac{1}{\sqrt{2x}}$

$f'(c) = 4f'(1)$

$\frac{1}{\sqrt{2c}} = 4 \cdot \frac{1}{\sqrt{2}}$

Square both sides $\rightarrow$ $\frac{1}{2c} = 16 \cdot \frac{1}{2}$

$1 = 16c$

(A) $\frac{1}{16}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) 1 (E) 32

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31. Let $f(x) = x \cdot g(h(x))$ where $g(4) = 2$, $g'(4) = 3$, $h(3) = 4$, and $h'(3) = -2$. Find $f'(3)$.

B

$f'(x) = 1 \cdot g(h(x)) + x \cdot g'(h(x)) \cdot h'(x)$

$f'(3) = g(4) + 3 \cdot g'(4) \cdot (-2) = 2 + 3(3)(-2) = 2 - 18$

(A) -18 (B) -16 (C) -7 (D) 7 (E) 11