Calculus

# 3.3 Differentiating Inverse Functions

**Notes**

Write your questions and thoughts here!

**Recall**:
A function's inverse is found by swapping the input ($x$) and output ($y$) values!

| Confusing Notation: | Reciprocal | | Inverse |
|---|---|---|---|
| | $x^{-1} =$ | or | $f^{-1}(x)$ **means** |

Three ways to say the same thing:
1. $g(x)$ is the inverse of $f(x)$
2. $g(x) = f^{-1}(x)$
3. $f\big(g(x)\big) = x$ and $g\big(f(x)\big) = x$

**Derivative of an Inverse Function:**

$$\frac{d}{dx}[f^{-1}(x)] =$$

The table below gives values of the differentiable functions $f$, $g$, and $f'$ at selected values of $x$. Let $g(x) = f^{-1}(x)$.

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1 | 3 | $-3$ |
| 2 | 1 | $-2$ |
| 3 | $-5$ | $-5$ |
| 4 | 0 | $-6$ |

1. What is the value of $g'(1)$?

2. Write an equation for the line tangent to $f^{-1}$ at $x = 1$.

3. Let $g$ be a differentiable function such that $g(12) = 4$, $g(3) = 6$, $g'(12) = -5$, and $g'(3) = -2$. The function $h$ is differentiable and $h(x) = g^{-1}(x)$ for all $x$. What is the value of $h'(6)$?

4. If $f(x) = 3x^3 + 1$ and $g$ is the inverse function of $f$, what is the value of $g'(25)$?

## 3.3 Differentiating Inverse Functions

Calculus

**Practice**

**For each problem, let $f$ and $g$ be differentiable functions where $g(x) = f^{-1}(x)$ for all $x$.**

1. $f(3) = -2, f(-2) = 4,$
   $f'(3) = 5$, and $f'(-2) = 1$.
   Find $g'(-2)$.

2. $f(1) = 5, f(2) = 4,$
   $f'(1) = -2$, and $f'(2) = -4$.
   Find $g'(5)$.

3. $f(6) = -2, f(-3) = 7,$
   $f'(6) = -1$, and $f'(-3) = 3$.
   Find $g'(7)$.

4. $f(-1) = 4, f(2) = -3,$
   $f'(-1) = -5$, and $f'(2) = 7$.
   Find $g'(-3)$.

**The table below gives values of the differentiable function $g$ and its derivative $g'$ at selected values of $x$. Let $h(x) = g^{-1}(x)$.**

| $x$ | $g(x)$ | $g'(x)$ |
|---|---|---|
| $-1$ | $-2$ | $-4$ |
| $-2$ | $-5$ | $-2$ |
| $-3$ | $-4$ | $-1$ |
| $-4$ | $-3$ | $-5$ |
| $-5$ | $-1$ | $-3$ |

5. Find $h'(-1)$

   Find the equation of the tangent line to $g^{-1}$ at $x = -1$.

6. $h'(-3)$

   Find the equation of the tangent line to $g^{-1}$ at $x = -3$.

7. $h'(-5)$

   Find the equation of the tangent line to $g^{-1}$ at $x = -5$.

| **$f$ and $g$ are differentiable functions. Use the table to answer the problems below. $f$ and $g$ are NOT inverses!** | | | | |
|---|---|---|---|---|
| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
| 1 | 5 | $-5$ | 4 | 5 |
| 2 | 1 | $-6$ | 3 | 3 |
| 3 | 6 | 4 | 1 | 6 |
| 4 | 2 | 9 | 6 | 1 |
| 5 | 3 | 1 | 1 | 2 |
| 6 | 4 | 2 | 2 | 4 |

8. $g^{-1}(4)$

9. $f^{-1}(5)$

10. $\frac{d}{dx}g^{-1}(3)$

11. $\frac{d}{dx}f^{-1}(1)$

12. Find the line tangent to the graph of $f^{-1}(x)$ at $x = 2$.

**For each function $g(x)$, its inverse $g^{-1}(x) = f(x)$. Evaluate the given derivative.**

13. $g(x) = \cos(x) + 3x^2$
$g\left(\frac{\pi}{2}\right) = \frac{3\pi}{4}$. Find $f'\left(\frac{3\pi}{4}\right)$

14. $g(x) = 2x^3 - x^2 - 5x$
$g(-2) = -10$. Find $f'(-10)$

15. $g(x) = \sqrt{8 - 2x}$. Find $f'(4)$?

16. $g(x) = x^3 - 7$. Find $f'(20)$?

17. $g(x) = \frac{5}{x+3}$. Find $f'\left(\frac{1}{2}\right)$?

18. The functions $f$ and $g$ are differentiable for all real numbers and $g$ is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of $x$. The function $h$ is given by $h(x) = f(g(x)) - 6$.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 6 | 4 | 2 | 5 |
| 2 | 9 | 2 | 3 | 1 |
| 3 | 10 | $-4$ | 4 | 2 |
| 4 | $-1$ | 3 | 6 | 7 |

(a) Explain why there must be a value $r$ for $1 < r < 3$ such that $h(r) = -5$.

(b) If $g^{-1}$ is the inverse function of $g$, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

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19. A function $h$ satisfies $h(3) = 5$ and $h'(3) = 7$. Which of the following statements about the inverse of $h$ must be true?

(A) $(h^{-1})'(5) = 3$ (B) $(h^{-1})'(7) = 3$ (C) $(h^{-1})'(5) = 7$

(D) $(h^{-1})'(5) = \frac{1}{7}$ (E) $(h^{-1})'(7) = \frac{1}{5}$