3.3 Differentiating Inverse Functions

Calculus

Solutions **Practice**

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

1. $f(3) = -2$, $f(-2) = 4$, $f'(3) = 5$, and $f'(-2) = 1$. Find $g'(-2)$.

$$\frac{1}{f'[f^{-1}(-2)]} = \frac{1}{f'(3)} = \boxed{\frac{1}{5}}$$

2. $f(1) = 5$, $f(2) = 4$, $f'(1) = -2$, and $f'(2) = -4$. Find $g'(5)$.

$$\frac{1}{f'[f^{-1}(5)]} = \frac{1}{f'(1)} = \boxed{-\frac{1}{2}}$$

3. $f(6) = -2$, $f(-3) = 7$, $f'(6) = -1$, and $f'(-3) = 3$. Find $g'(7)$.

$$\frac{1}{f'[f^{-1}(7)]} = \frac{1}{f'(-3)} = \boxed{\frac{1}{3}}$$

4. $f(-1) = 4$, $f(2) = -3$, $f'(-1) = -5$, and $f'(2) = 7$. Find $g'(-3)$.

$$\frac{1}{f'[f^{-1}(-3)]} = \frac{1}{f'(2)} = \boxed{\frac{1}{7}}$$

The table below gives values of the differentiable function g and its derivative g' at selected values of x. Let $h(x) = g^{-1}(x)$.

x	$g(x)$	$g'(x)$
-1	-2	-4
-2	-5	-2
-3	-4	-1
-4	-3	-5
-5	-1	-3

5. Find $h'(-1)$

$$\frac{1}{g'(g^{-1}(-1))} = \frac{1}{g'(-5)} = \boxed{-\frac{1}{3}}$$

Find the equation of the tangent line to g^{-1} at $x = -1$.

$$\boxed{y+5 = -\tfrac{1}{3}(x+1)}$$

6. $h'(-3)$

$$\frac{1}{g'(g^{-1}(-3))} = \frac{1}{g'(-4)} = \boxed{-\frac{1}{5}}$$

Find the equation of the tangent line to g^{-1} at $x = -3$.

$$\boxed{y+4 = -\tfrac{1}{5}(x+3)}$$

7. $h'(-5)$

$$\frac{1}{g'(g^{-1}(-5))} = \frac{1}{g'(-2)} = \boxed{-\frac{1}{2}}$$

Find the equation of the tangent line to g^{-1} at $x = -5$.

$$\boxed{y+2 = -\tfrac{1}{2}(x+5)}$$

f and g are differentiable functions. Use the table to answer the problems below. f and g are NOT inverses!				
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-5	4	5
2	1	-6	3	3
3	6	4	1	6
4	2	9	6	1
5	3	1	1	2
6	4	2	2	4

8. $g^{-1}(4)$

$\boxed{1}$

9. $f^{-1}(5)$

$\boxed{1}$

10. $\frac{d}{dx}g^{-1}(3)$

$\frac{1}{g'(g^{-1}(3))} = \frac{1}{g'(2)} = \boxed{\frac{1}{3}}$

11. $\frac{d}{dx}f^{-1}(1)$

$\frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(2)} = \boxed{-\frac{1}{6}}$

12. Find the line tangent to the graph of $f^{-1}(x)$ at $x = 2$.

$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(4)} = \frac{1}{9}$

$\boxed{y-4 = \frac{1}{9}(x-2)}$

For each function $g(x)$, its inverse $g^{-1}(x) = f(x)$. Evaluate the given derivative.

13. $g(x) = \cos(x) + 3x^2$
$g\left(\frac{\pi}{2}\right) = \frac{3\pi}{4}$. Find $f'\left(\frac{3\pi}{4}\right)$

$g'(x) = -\sin x + 6x$

$g'\left(\frac{\pi}{2}\right) = -1 + 3\pi$

$\frac{1}{g'(g^{-1}(\frac{3\pi}{4}))} = \frac{1}{g'(\frac{\pi}{2})} = \boxed{\frac{1}{3\pi - 1}}$

14. $g(x) = 2x^3 - x^2 - 5x$
$g(-2) = -10$. Find $f'(-10)$

$g'(x) = 6x^2 - 2x - 5$

$g'(-2) = 24 + 4 - 5$

$= 23$

$f'(-10) = \frac{1}{g'(g^{-1}(-10))} = \frac{1}{g'(-2)} = \boxed{\frac{1}{23}}$

15. $g(x) = \sqrt{8 - 2x}$. Find $f'(4)$?

$g'(x) = \frac{-2}{2\sqrt{8-2x}} = \frac{-1}{\sqrt{8-2x}}$

$4 = \sqrt{8-2x}$

$16 = 8 - 2x$

$-4 = x \rightarrow g^{-1}(4) = -4$

$\frac{1}{g'(g^{-1}(4))} = \frac{1}{g'(-4)} = \frac{1}{\frac{-1}{\sqrt{16}}}$

$\boxed{-4}$

16. $g(x) = x^3 - 7$. Find $f'(20)$?

$20 = x^3 - 7$

$27 = x^3$

$3 = x \rightarrow g^{-1}(20) = 3$

$\frac{1}{g'(g^{-1}(20))} = \frac{1}{g'(3)}$

$g'(x) = 3x^2$

$g'(3) = 27$

$\boxed{\frac{1}{27}}$

17. $g(x) = \frac{5}{x+3}$. Find $f'\left(\frac{1}{2}\right)$?

$\frac{1}{2} = \frac{5}{x+3}$

$x + 3 = 10$

$x = 7 \rightarrow g^{-1}(\frac{1}{2}) = 7$

$\frac{1}{g'(g^{-1}(\frac{1}{2}))} = \frac{1}{g'(7)} \rightarrow \frac{1}{-\frac{1}{20}}$

$g'(x) = -\frac{5}{(x+3)^2}$

$g'(7) = -\frac{5}{100} = -\frac{1}{20}$

$\boxed{-20}$

18. The functions f and g are differentiable for all real numbers and g is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

According to the IVT, there is a value r such that $h(r) = -5$ and $1 \leq r \leq 3$.

(b) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$\frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$

$\boxed{y - 1 = \frac{1}{5}(x - 2)}$

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19. A function h satisfies $h(3) = 5$ and $h'(3) = 7$. Which of the following statements about the inverse of h must be true?

$h^{-1}(5) = 3$

$\frac{d}{dx} h^{-1}(5) = \frac{1}{h'(h^{-1}(5))} = \frac{1}{h'(3)} = \frac{1}{7}$

(A) $(h^{-1})'(5) = 3$

(B) $(h^{-1})'(7) = 3$

(C) $(h^{-1})'(5) = 7$

(D) $(h^{-1})'(5) = \frac{1}{7}$

(E) $(h^{-1})'(7) = \frac{1}{5}$