3.6 Calculating Higher-Order Derivatives

Calculus

Solutions **Practice**

Find $\frac{d^2y}{dx^2}$ based on the given information.

1. $y = \sin x + \ln(5x)$

$y' = \cos x + \frac{1}{x}$

$\boxed{y'' = -\sin x - \frac{1}{x^2}}$

2. $y = e^x x$

$y' = e^x x + e^x(1)$

$y'' = e^x x + e^x(1) + e^x$

$\boxed{y'' = e^x x + 2e^x}$

3. $y = \sin^2 x$

$y' = 2\sin x \cos x$

$\boxed{y'' = 2\cos^2 x - 2\sin^2 x}$

5. $\frac{dy}{dx} = y^2 + 2x - 1$

$$\frac{d^2y}{dx^2} = 2y\frac{dy}{dx} + 2$$
$$= 2y(y^2+2x-1)+2$$
$$\boxed{= 2y^3+4xy-2y+2}$$

6. $\frac{dy}{dx} = \frac{1}{y} - 3x$

$$\frac{d^2y}{dx^2} = -\frac{1}{y^2}\frac{dy}{dx} - 3$$
$$= -\frac{1}{y^2}\left(\frac{1}{y}-3x\right)-3$$
$$\boxed{= -\frac{1}{y^3} + \frac{3x}{y^2} - 3}$$

7. $\frac{dy}{dx} = xy^2$

$$\frac{d^2y}{dx} = y^2 + 2xy\frac{dy}{dx}$$
$$= y^2 + 2xy(xy^2)$$
$$\boxed{= y^2 + 2x^2y^3}$$

8. $\sin(x+y) = 2x$

$$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 2$$
$$1+\frac{dy}{dx} = 2\sec(x+y)$$
$$\frac{dy}{dx} = 2\sec(x+y)-1$$
$$\frac{d^2y}{dx^2} = 2\sec(x+y)\tan(x+y)\cdot\left(1+\frac{dy}{dx}\right)$$
$$= 2\sec(x+y)\tan(x+y)\left(1+2\sec(x+y)-1\right)$$
$$\boxed{= 4\sec^2(x+y)\tan(x+y)}$$

9. $e^x = y^3 + 1$

$$e^x = 3y^2\frac{dy}{dx}$$
$$\frac{e^x}{3y^2} = \frac{dy}{dx}$$
$$\frac{d^2y}{dx^2} = \frac{e^x(3y^2) - 6ye^x\left(\frac{dy}{dx}\right)}{9y^4}$$
$$\frac{3y^2e^x - 6ye^x\left(\frac{e^x}{3y^2}\right)}{9y^4}$$
$$\boxed{\frac{3y^2e^x - \frac{2e^{2x}}{y}}{9y^4}}$$

10. $\ln(y) = 5x + 3$

$$\frac{1}{y}\frac{dy}{dx} = 5$$
$$\frac{dy}{dx} = 5y$$
$$\frac{d^2y}{dx^2} = 5\frac{dy}{dx}$$
$$= 5(5y)$$
$$\boxed{= 25y}$$

Evaluate the 2nd derivative at the given point.

11. If $f(x) = -3x^3 + 4x^{-2}$, find $f''(-2)$.

$$f'(x) = -9x^2 - 8x^{-3}$$
$$f''(x) = -18x + 24x^{-4}$$
$$f''(-2) = -18(-2) + \frac{24}{(-2)^4}$$
$$= 36 + \frac{24}{16}$$
$$= 36 + \frac{3}{2} = \boxed{\frac{75}{2}}$$

12. If $f(x) = x\ln x$, find $f''(1)$.

$$f'(x) = \ln x + x\left(\frac{1}{x}\right) = \ln x + 1$$
$$f''(x) = \frac{1}{x}$$
$$f''(1) = \boxed{1}$$

13. If $f(x) = 3\sqrt{x} - \frac{32}{x}$, find $f''(4)$.

$f'(x) = \frac{3}{2}x^{-1/2} + 32x^{-2}$

$f''(x) = -\frac{3}{4}x^{-3/2} - 64x^{-3}$

$f''(4) = -\frac{3}{4(8)} - \frac{64}{64}$

$= -\frac{3}{32} - 1$

$= \boxed{-35/32}$

14. If $\frac{dy}{dx} = 3\cos y + 5x$, find $\frac{d^2y}{dx^2}$ at $\left(2, \frac{\pi}{2}\right)$

$\frac{dy}{dx}_{(2,\pi/2)} = 3\cos(\pi/2) + 5(2) = 10$

$\frac{d^2y}{dx^2} = -3\sin y \frac{dy}{dx} + 5$

$\frac{d^2y}{dx^2}_{(2,\pi/2)} = -3\sin(\pi/2)(10) + 5$

$= -30 + 5$

$\boxed{-25}$

15. If $\frac{dy}{dx} = \frac{4-x}{2y-3}$, find $\frac{d^2y}{dx^2}$ at $(-1, 2)$

$\frac{dy}{dx}_{(-1,2)} = \frac{4+1}{4-3} = 5$

$\frac{d^2y}{dx^2} = \frac{(-1)(2y-3) - (4-x)(2)\frac{dy}{dx}}{(2y-3)^2}$

$\frac{d^2y}{dx^2}_{(-1,2)} = \frac{-1(4-3) - (4+1)(2)(5)}{(4-3)^2}$

$-1 - 50 = \boxed{-51}$

16. If $\frac{dy}{dx} = \ln x\, e^y$, find $\frac{d^2y}{dx^2}$ at $(e, 1)$

$\frac{dy}{dx}_{(e,1)} = \ln e \cdot e^1 = e$

$\frac{d^2y}{dx^2} = \frac{1}{x}e^y + \ln x \cdot e^y \frac{dy}{dx}$

$\frac{d^2y}{dx^2}_{(e,1)} = \frac{1}{e} \cdot e + \ln e \cdot e(e)$

$\boxed{1 + e^2}$

Find the derivatives of the following.

17. $f(x) = 3x^7 - 4x^3 + 5x$

$f'(x) = 21x^6 - 12x^2 + 5$

$f''(x) = 126x^5 - 24x$

$f'''(x) = 630x^4 - 24$

$f^{(4)}(x) = 2520x^3$

18. $y = 4\sqrt{x}$

$4x^{1/2}$

$\frac{dy}{dx} = 2x^{-1/2}$

$\frac{d^2y}{dx^2} = -x^{-3/2}$

19. $y = \frac{1}{x^3} - \frac{1}{2}x^4 = x^{-3} - \frac{1}{2}x^4$

$y' = -3x^{-4} - 2x^3$

$y'' = 12x^{-5} - 6x^2$

$y''' = -60x^{-6} - 12x$

Given $f(x) = 3x^2 - x + 2$, $g(x) = \frac{1}{x^3}$, and $h(x) = \sqrt{x}$, find the following.

20. $f''(2) =$

$f'(x) = 6x - 1$

$f''(x) = 6$

$f''(2) = \boxed{6}$

21. $g'''(-3) =$

$g'(x) = -3x^{-4}$

$g''(x) = 12x^{-5}$

$g'''(x) = -\frac{60}{x^6}$

$g'''(-3) = \frac{-60}{(-3)^6} = \boxed{-\frac{20}{243}}$

22. $2h''(4) =$

$h'(x) = \frac{1}{2}x^{-1/2}$

$h''(x) = -\frac{1}{4}x^{-3/2}$

$2h''(4) = 2\left[-\frac{1}{4(8)}\right]$

$= \boxed{-\frac{1}{16}}$

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Test Prep

23. If $f(x) = \left(1+\frac{x}{20}\right)^5$, find $f''(40)$.

$f'(x) = 5\left(1+\frac{x}{20}\right)^4 \cdot \frac{1}{20} = \frac{1}{4}\left(1+\frac{x}{20}\right)^4$

$f''(x) = \left(1+\frac{x}{20}\right)^3 \cdot \frac{1}{20}$

$f''(40) = (1+2)^3 \cdot \frac{1}{20}$

(A) 0.068

(B) 1.350

(C) 5.400

(D) 6.750

(E) 540.000

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24. A curve given by the equation $x^3 + xy = 8$ has slope given by $\frac{dy}{dx} = \frac{-3x^2-y}{x}$. The value of $\frac{d^2y}{dx^2}$ at the point where $x = 2$ is

Find y when $x=2$.

$2^3+2y=8$

$2y=0$

$y=0$

$(2,0)$

$\frac{dy}{dx}\Big|_{(2,0)} = \frac{-3(4)}{2} = -6$

$\frac{d^2y}{dx^2} = \frac{\left(-6x-\frac{dy}{dx}\right)x - (-3x^2-y)}{x^2}$

$\frac{d^2y}{dx^2}\Big|_{(2,0)} = \frac{(-6\cdot(2)-(-6))(2) - (-3(4)-0)}{4}$

(A) -6 (B) -3 (C) 0 (D) 4 (E) undefined

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25. If $y = xe^x$, then $\frac{d^ny}{dx^n} =$

$\frac{dy}{dx} = e^x + xe^x = e^x(1+x)$

$\frac{d^2y}{dx^2} = e^x + e^x + xe^x = e^x(2+x)$

$\frac{d^3y}{dx^3} = e^x + e^x + e^x + xe^x = e^x(3+x)$

(A) e^x (B) e^{nx} (C) $(x+n)e^x$ (D) x^ne^x (E) $(x+n^2)e^x$

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26. Let g be the function given by $g(x) = \cos(-x) - \sin x + 6$. Which of the following statements is true for $y = g(x)$?

$g'(x) = -\sin(-x)\cdot(-1) - \cos x = \sin(-x) - \cos x$

$g''(x) = -\cos(-x) + \sin x$

$g^{(3)}(x) = -\sin(-x) + \cos x$

$g^{(4)}(x) = \underbrace{\cos(-x) - \sin x}_{\text{compare to } g(x)}$

(A) $y - 6 = \frac{d^4y}{dx^4}$

(B) $g^{(4)}(x) = \left(g''(x)\right)^2$

(C) $g''(x) - 6 = g(x)$

(D) $y'' = \cos(-x) - \sin x$