## 4.2 Position, Velocity, and Acceleration

Calculus

Solutions **Practice**

1. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function

$$s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 5$$

where $s$ is measured in meters and $t$ is measured in seconds.

a. Find the instantaneous velocity at any time $t$.

$v(t) = t^2 - 6t + 8$

b. When is the particle at rest?

$v(t) = 0$

$(t-4)(t-2) = 0$

$t = 2$ and $t = 4$ seconds

c. Find the acceleration of the particle at any time $t$.

$a(t) = 2t - 6$

d. What is the displacement of the particle for the first 3 seconds?

$s(0) = -5$

$s(3) = 9 - 27 + 24 - 5 = 1$

$s(3) - s(0) =$ 6 meters

2. A ball is dropped off a 1200 ft cliff. The height of the ball over time is modeled by the function $h(t) = 1200 - 16t^2$ where $h$ is the height of the ball and $t$ is time in seconds.

a. Find $h'(3)$. Explain what it means.

$h'(t) = -32t$

$h'(3) = -96$

At 3 seconds, the ball is falling at a rate of 96 feet / second.

b. Find $h''(3)$. Explain what it means.

$h''(t) = -32$

$h''(3) = -32$

At 3 seconds, the rate the ball is falling is increasing by 32 feet / second per second.

3. The position, in meters, of a body at time $t$ sec is $s(t) = t^3 - 6t^2 + 9t$. Find the body's acceleration each time the velocity is zero.

$v(t) = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = (t-3)(t-1)$

$t = 1 \quad t = 3$

$a(t) = 6t - 12$

$a(1) = 6 - 12 \qquad a(3) = 18 - 12$

$a(1) = -6$ m/sec$^2$ $\quad a(3) = 6$ m/sec$^2$

4. A particle $P$ moves on the number line. The graph $s = f(t)$ shows the position of $P$ as a function of time $t$.

a. When is $P$ moving to the left? → $s'(t) < 0$

(2, 3) and (5, 6)

b. When is $P$ moving to the right? → $s'(t) > 0$

(0, 1)

c. When is $P$ at rest? → $s'(t) = 0$

(1, 2) and (3, 5)

d. Graph the particle's velocity where defined.

5. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

a. When does the body reverse direction?

$t=2,\ t\approx 7.8$ Velocity changes sign

b. When is the body moving at a constant speed?

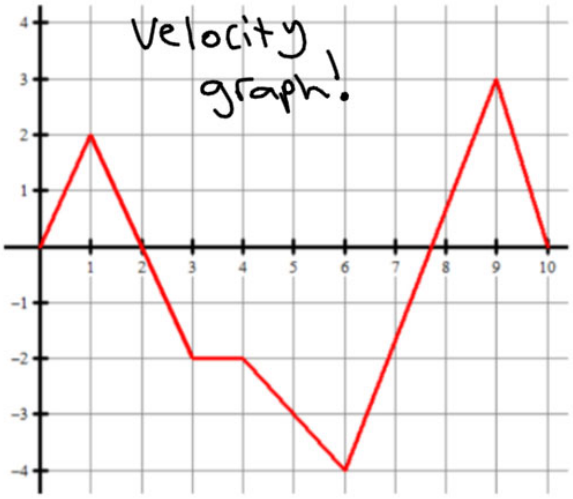
$(3,4)$

c. What is the body's maximum speed?

4 meters per second

d. What time interval(s) is the body speeding up?

$(0,1)\ (2,3)\ (4,6)\ (7.8,9)$




6. A rock thrown vertically upward from the surface of the moon at a velocity of 32 meters per second reaches a height of $s(t) = 32t - 0.8t^2$ meters in $t$ seconds.

a. Find the rock's velocity and acceleration as functions of time.

$v(t) = 32 - 1.6t$

$a(t) = -1.6$

b. How long did it take the rock to reach its highest point?

at rest means $v(t)=0$.

$32 - 1.6t = 0$

$t = 20$ seconds

7. The data in the table gives selected values for the velocity, in meters per minute, of a particle moving along the $x$-axis. The velocity $v$ is a differentiable function of time $t$.

| **Time $t$** | 0 | 2 | 5 | 6 | 8 | 12 |
|---|---|---|---|---|---|---|
| **Velocity $v(t)$** | -3 | 2 | 3 | 5 | 7 | 5 |

a. At $t = 0$, is the particle moving to the right or left? Justify.

Left because $v(0)<0$.

b. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Justify.

Yes, between 0 and 2 minutes. By the Intermediate Value Theorem (IVT), v(t) must equal zero in that interval.

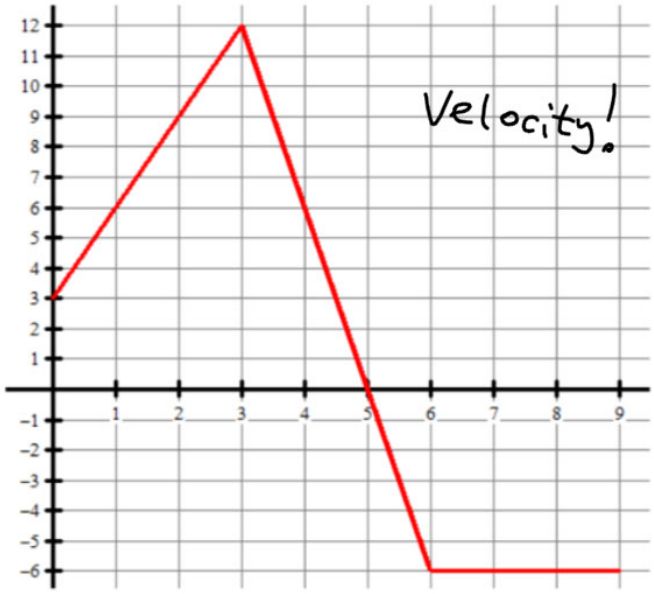
c. Use the data from the table to approximate $v'(10)$.

$\frac{v(12)-v(8)}{12-8} = \frac{-2}{4} = -\frac{1}{2}$

d. Explain the meaning of $v'(10)$ in terms of the particle motion.

At 10 minutes, the particle's velocity is decreasing by 0.5 meters per minute$^2$.

8. The graph represents the velocity, in feet per second, of a particle moving along the $x$-axis over the time interval from $t = 0$ to $t = 9$ seconds.




a. At $t = 4$, is the particle moving to the right or left? Justify.

Right because $v(4) > 0$.

b. Over what time interval is the particle moving left? Justify.

$(5, 9)$ because $v(t) < 0$.

c. At $t = 4$, is the acceleration positive or negative? Justify.

Negative. $v'(4) < 0$.

d. What is the average acceleration of the particle over the interval $2 \le t \le 4$ ? Show the computations and label your answer.

$$\frac{v(4) - v(2)}{4 - 2} = \frac{6 - 9}{2} = -\frac{3}{2} \text{ ft/sec}^2$$

e. At what time $t$ in the given interval is the particle furthest to the right. Justify.

t=5. The particle travels right from (0, 5) seconds, then travels left for (5, 9) seconds.

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9. A particle moves along the $x$-axis so that at time $t$ its position is given by

$$x(t) = t^3 - 6t^2 + 9t + 11$$

where $t$ is measured in seconds and $x$ is measured in feet.

a. At $t = 0$, is the particle moving to the right or left? Justify.

$x'(t) = 3t^2 - 12t + 9$

$x'(0) = 9$

Right because $x'(0) > 0$

b. At $t = 1$, is the velocity of the particle increasing or decreasing? Justify.

$x''(t) = 6t - 12$

$x''(1) = 6 - 12 = -6$

Decreasing because $x''(1) < 0$.

c. What is the displacement over the first 6 seconds?

$x(0) = 11$

$x(6) = 65$

$x(6) - x(0) =$ 54 feet

No test prep. These questions are already like exam type problems.