4.4 Introduction to Related Rates

Notes

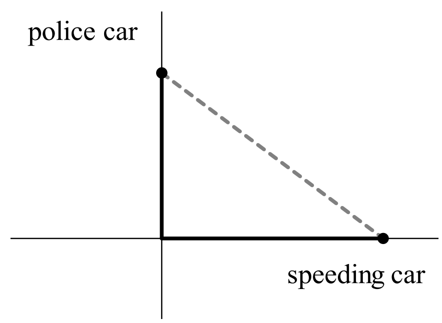
Write your questions and thoughts here!

Many variables have a familiar relationship. Or in other words, a way variables are "related" to each other. A great example is the Pythagorean Theorem.

Let's say a triangle's dimensions are changing. This means a dimension (a, b, or c) has a rate of change, and this rate is "related" to another dimension's rate of change.

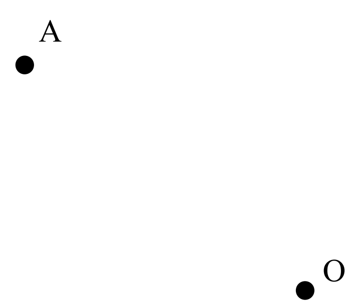
Differentiate your relationship with respect to one specific variable. This is like implicit differentiation. Usually it is with respect to time, but not always.

1. A police car, approaching a right-angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. Set up a relationship, then find an equation that shows the related rates of the vehicles with respect to time.



Some dimensions are constants.

An airplane (point A) is flying on a horizontal path that will take it directly over an observer (point O). The airplane maintains a constant altitude. Relate the rates of change of the distance between the observer and the airplane and the horizontal distance between the two.



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Calculus

Practice

Find a relationship between the given rates of change by doing the following.
1. **Set up a relationship using variables for the situation.**
2. **Differentiate with respect to time t.**

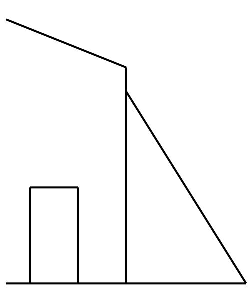
1. An ice cube is melting. Relate the volume's rate of change with the edges' rate of change.

2. The width and length of a rectangle are increasing. Relate these rates of change with the rate of change of the area of the rectangle.

3. A spherical balloon is expanding. Relate the rate of change of surface area with the rate of change of the radius of the balloon. $A = 4\pi r^2$.

4. The water level is dropping in a cylindrical tank because of a small leak in the tank. Relate the rates of change between the water level and the volume of the water. The volume is modeled by $V = \pi r^2 h$.

5. Mr. Brust is using a ladder to paint his house. The ladder is leaning against the house when Mr. Sullivan decides to pull the base of the ladder away from the house. Set up a relationship of the rates of change of how high the top of the ladder is and the distance from the bottom of the ladder to the house.



6. A rocket is rising vertically. An observer on the ground is standing a certain distance from the rocket's launch point. As the observer watches the rocket, the angle of elevation is increasing. Relate the rates of change of the angle of elevation with the speed of the rocket.

7. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock. The pulley is higher than the boat's bow by several feet. Relate the rate that the rope is hauled in with how fast the boat is approaching the dock.

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Test Prep

8. Let a and b be functions of time t such that the sum of a and three times b is constant. Which of the following equations describes the relationship between the rate of change of a with respect to time and the rate of change of b with respect to time?

(A) $\frac{da}{dt} = 3\frac{db}{dt}$

(B) $\frac{da}{dt} = -3\frac{db}{dt}$

(C) $3\frac{da}{dt} + \frac{db}{dt} = 0$

(D) $\frac{da}{dt} + 3\frac{db}{dt} = F$, where F is a function of t.

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9. The number of minutes M that it takes to make a calculus lesson and the number of lessons L that are made per week satisfy the relationship $L = \frac{k}{M}$, where k is a constant. Which of the following best describes the relationship between the rate of change, with respect to time t, of L and the rate of change, with respect to time t, of M?

(A) $\frac{dL}{dt} = \frac{k}{\left(\frac{dM}{dt}\right)}$

(B) $\frac{dL}{dt} = \frac{-k}{\left(\frac{dM}{dt}\right)}$

(C) $\frac{dL}{dt} = \frac{k}{M^2}\left(\frac{dM}{dt}\right)$

(D) $\frac{dL}{dt} = \frac{-k}{M^2}\left(\frac{dM}{dt}\right)$