Calculus

4.5 Solving Related Rates

Notes

Write your questions and thoughts here!

1. If $y^2 = 3x^4 + 6x$, find $\frac{dy}{dt}$ when $x = 1$, and $\frac{dx}{dt} = 2$.

Guidelines to solving related rate problems

1. Draw a picture.
2. Make a list of all known and unknown rates and quantities.
3. Relate the variables in an equation.
4. Differentiate with respect to time.
5. Substitute the known quantities and rates and solve.

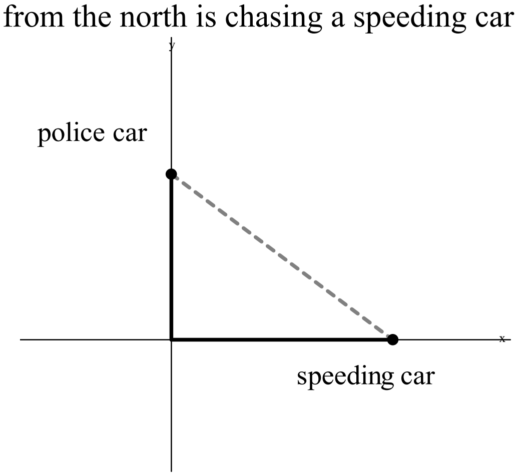
IMPORTANT: Substituting a non-constant quantity before differentiating is not allowed!

Rectangle Example

The width of a rectangle is increasing at a rate of 2 cm/sec and its length is increasing at a rate of 3 cm/sec. At what rate is the area of the rectangle increasing when its width is 4 cm and its length is 5 cm?

Triangle Example

A police car, approaching a right-angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. When the police car is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with a radar gun that the distance between them and the car is increasing at 20 mph. If the police car is moving at 60 mph at the instant of measurement, what is the speed of the car?



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Practice

1. If $y = 3x^4 + 6x$, find $\frac{dy}{dt}$ when $x = 1$, and $\frac{dx}{dt} = -3$.

2. If $g = 5h - h^5$, find $\frac{dg}{dt}$ when $h = 2$, and $\frac{dh}{dt} = 3$.

3. If $x^2 + y^2 = z^2$, find $\frac{dy}{dt}$ when $x = 3$, $y = 4$, $\frac{dx}{dt} = -1$, and $\frac{dz}{dt} = 5$.

4. If $A = \frac{1}{2}bh$, find $\frac{dA}{dt}$ when $b = 7$, $h = 6$, $\frac{db}{dt} = 2$, and $\frac{dh}{dt} = -3$.

5. An ice cube is melting at a rate of 5 cubic cm per hour. At what rate is the edge of the cube changing when the edge of the cube is 3 cm.

Answer: $-\frac{5}{27}$ cm/hour

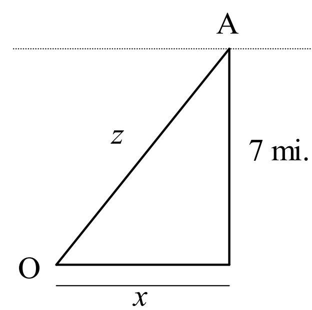
6. A circular pool of water is expanding at the rate of 16π in^2 / sec. At what rate is the radius expanding when the radius is 4 inches?

Answer: 2 inches/sec

7. A spherical balloon is expanding at a rate of 60π in^3 / sec. How fast is the surface area of the balloon expanding when the radius of the balloon is 4 inches? $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

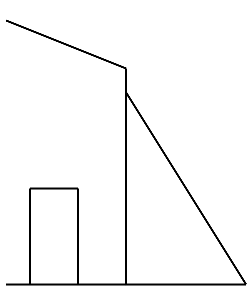
Answer: 30π in^2/sec

8. An airplane (pt. A) is flying 600 mph on a horizontal path that will take it directly over an observer (pt. O). The airplane maintains a constant altitude of 7 miles (see figure). What is the rate of change of the distance between the observer and the airplane when $x = 5$ miles?



Answer: $-\frac{3000}{\sqrt{74}}$ mph

9. Mr. Brust is using a ladder to paint his house. The 17-ft ladder is leaning against the house when Mr. Kelly decides to pull the base of the ladder away from the house at a rate of 3 ft./sec. How fast is the top of the ladder moving down the side of the house when it is 8 ft. above the ground? Indicate units of measure.



Answer: $-\frac{45}{8}$ feet/sec

10. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock. The pulley is 7 feet higher than the boat's bow. If the rope is hauled in at a rate of 4 feet/sec, how fast is the boat approaching the dock when 25 feet of rope is out?

Answer: $\frac{25}{6}$ feet/sec

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Test Prep

11. The base of a triangle is decreasing at a constant rate of 0.2 cm/sec and the height is increasing at 0.1 cm/sec. If the area is increasing, which answer best describes the constraints on the height h at the instant when the base is 3 centimeters?

(a) $h > 3$ (b) $h < 1$ (c) $h > 1.5$ (d) $h < 1.5$ (e) $h > 2$