4.5 Solving Related Rates

Calculus

Solutions **Practice**

1. If $y = 3x^4 + 6x$, find $\frac{dy}{dt}$ when $x = 1$, and $\frac{dx}{dt} = -3$.

$$\frac{dy}{dt} = 12x^3\frac{dx}{dt} + 6\frac{dx}{dt}$$
$$\frac{dy}{dt} = 12(1)^3(-3) + 6(-3)$$
$$\frac{dy}{dt} = -36 - 18$$
$$\boxed{\frac{dy}{dt} = -54}$$

2. If $g = 5h - h^5$, find $\frac{dg}{dt}$ when $h = 2$, and $\frac{dh}{dt} = 3$.

$$\frac{dg}{dt} = 5\frac{dh}{dt} - 5h^4\frac{dh}{dt}$$
$$\frac{dg}{dt} = 5(3) - 5(2)^4(3)$$
$$\frac{dg}{dt} = 15 - 240$$
$$\boxed{\frac{dg}{dt} = -225}$$

3. If $x^2 + y^2 = z^2$, find $\frac{dy}{dt}$ when $x = 3$, $y = 4$, $\frac{dx}{dt} = -1$, and $\frac{dz}{dt} = 5$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

?

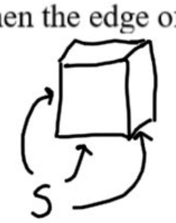
$$3^2 + 4^2 = z^2$$
$$25 = z^2$$
$$5 = z$$

$$2(3)(-1) + 2(4)\frac{dy}{dt} = 2(5)(5)$$
$$-6 + 8\frac{dy}{dt} = 50$$
$$\boxed{\frac{dy}{dt} = 7}$$

4. If $A = \frac{1}{2}bh$, find $\frac{dA}{dt}$ when $b = 7$, $h = 6$, $\frac{db}{dt} = 2$, and $\frac{dh}{dt} = -3$.

$$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt}h + \frac{1}{2}b\frac{dh}{dt}$$
$$\frac{dA}{dt} = \frac{1}{2}(2)(6) + \frac{1}{2}(7)(-3)$$
$$= 6 - \frac{21}{2}$$
$$= \boxed{-\frac{9}{2}}$$

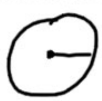
5. An ice cube is melting at a rate of 5 cubic cm per hour. At what rate is the edge of the cube changing when the edge of the cube is 3 cm.



$$\frac{dV}{dt} = -5$$
$$\frac{ds}{dt} = ?$$
$$s = 3$$

$$V = s^3$$
$$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$$
$$-5 = 3(3)^2\frac{ds}{dt}$$
$$\boxed{\frac{ds}{dt} = -\frac{5}{27} \text{ cm/hour}}$$

6. A circular pool of water is expanding at the rate of 16π in^2 / sec. At what rate is the radius expanding when the radius is 4 inches?



$$\frac{dA}{dt} = 16\pi$$
$$\frac{dr}{dt} = ?$$
$$r = 4$$

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$
$$16\pi = 2\pi(4)\frac{dr}{dt}$$
$$\boxed{\frac{dr}{dt} = 2 \text{ in/sec}}$$

7. A spherical balloon is expanding at a rate of 60π in^3 / sec. How fast is the surface area of the balloon expanding when the radius of the balloon is 4 inches? $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

$\frac{dV}{dt} = 60\pi$

$\frac{dA}{dt} = ?$

$r = 4$

$A = 4\pi r^2$?

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$60\pi = 4\pi (4)^2 \frac{dr}{dt}$

$\frac{15}{16} = \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi(4)\left(\frac{15}{16}\right)$

$\boxed{\frac{dA}{dt} = 30\pi \text{ in}^2/\text{sec}}$

8. An airplane (pt. A) is flying 600 mph on a horizontal path that will take it directly over an observer (pt. O). The airplane maintains a constant altitude of 7 miles (see figure). What is the rate of change of the distance between the observer and the airplane when $x = 5$ miles?

$\frac{dx}{dt} = -600$

$\frac{dy}{dt} = 0$

$\frac{dz}{dt} = ?$

$x = 5$

$5^2 + 7^2 = z^2$

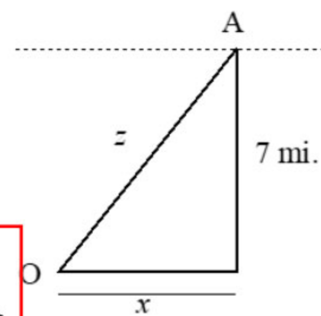
$\sqrt{74} = z$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

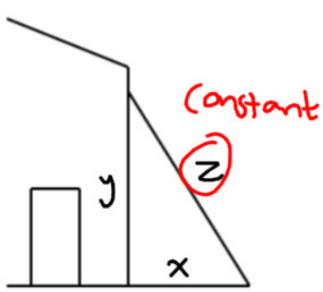
$2(5)(-600) + 0 = 2\sqrt{74}\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = -\frac{3000}{\sqrt{74}} \text{ mph}}$



constant!

9. Mr. Brust is using a ladder to paint his house. The 17-ft ladder is leaning against the house when Mr. Kelly decides to pull the base of the ladder away from the house at a rate of 3 ft./sec. How fast is the top of the ladder moving down the side of the house when it is 8 ft. above the ground? Indicate units of measure.



$z = 17$

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = ?$

$y = 8$

$x^2 + 8^2 = 17^2$

$x = 15$

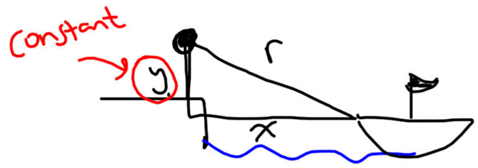
$x^2 + y^2 = 17^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(15)(3) + 2(8)\frac{dy}{dt} = 0$

$\boxed{\frac{dy}{dt} = -\frac{45}{8} \text{ ft/sec}}$

10. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock. The pulley is 7 feet higher than the boat's bow. If the rope is hauled in at a rate of 4 feet/sec, how fast is the boat approaching the dock when 25 feet of rope is out?



$y = 7$

$\frac{dr}{dt} = -4$

$\frac{dx}{dt} = ?$

$r = 25$

$x^2 + 7^2 = 25^2$

$x = 24$

$x^2 + 7^2 = r^2$

$2x\frac{dx}{dt} + 0 = 2r\frac{dr}{dt}$

$2(24)\frac{dx}{dt} = 2(25)(-4)$

$$\boxed{\frac{dx}{dt} = -\frac{25}{6} \text{ ft/sec}}$$

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Test Prep

11. The base of a triangle is decreasing at a constant rate of 0.2 cm/sec and the height is increasing at 0.1 cm/sec. If the area is increasing, which answer best describes the constraints on the height h at the instant when the base is 3 centimeters?

$A = \frac{1}{2}bh$

$\frac{dh}{dt} = 0.1$

$\frac{db}{dt} = -0.2$

$\frac{dA}{dt} > 0$

$b = 3$

$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt}h + \frac{1}{2}b\frac{dh}{dt}$

$\frac{1}{2}(-0.2)h + \frac{1}{2}(3)(0.1) > 0$

$-0.1h > -0.15$

(a) $h > 3$ (b) $h < 1$ (c) $h > 1.5$ (d) $h < 1.5$ (e) $h > 2$