Calculus

4.7 L'Hospital's Rule

Notes

Write your questions and thoughts here!

Recall: When evaluating limits, first try direct substitution! $\lim_{x\to 3}\frac{2x-5}{x}=$

1. $\lim_{x\to 2}\frac{x^2-7x+10}{x-2}=$

L'Hospital's Rule:

Suppose $f(a)=0$ and $g(a)=0$ and $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$ or $\frac{\infty}{\infty}$. L'Hopital's Rule allows you to apply the following:

Evaluate each limit. Use L'Hospital's when possible.

2. $\lim_{x\to 2}\frac{x-2}{3x^3-6x^2+x-2}$

3. $\lim_{x\to 0}\frac{\sin(6x)}{x}$

4. $\lim_{x\to 0}\frac{1-\cos(x)}{x^2}$

5. $\lim_{x\to \infty}\frac{2x^2}{e^{2x}}$

L'HOSPITAL'S IS NOT THE QUOTIENT RULE!!

6. $\frac{d}{dx}\frac{\sin(6x)}{x}$

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Practice

Find the following. Use L'Hôpital's when possible.

1. $\lim_{x\to 1}\frac{x-1}{x^2-3x+2}$

2. $\lim_{x\to -5}\frac{x^2-2x-35}{x+5}$

3. $\lim_{x\to 0}\frac{4x}{\ln(x+1)}$

4. $\lim_{x\to 0}\frac{x-1}{x^2-3x+2}$

5. $\lim_{x\to 1}\frac{2(x^2-1)}{\ln x^2}$

6. $\frac{d}{dx}\frac{6x^2+x}{\sin(x)}$

7. $\lim_{x\to 0}\frac{2x^2}{e^x-1-x}$

8. $\lim_{x\to 0}\frac{2x^2}{1-\cos(4x)}$

9. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$

10. $\lim_{x\to -3}\frac{x-1}{x^2+7x+10}$

11. $\lim_{x\to \infty}\frac{e^{2x}}{2x^2}$

12. $\lim_{x\to 0}\frac{\cos x-1}{x^2}$

13. $\frac{d}{dx}\frac{6x^2+x}{x+1}$

14. $\lim_{x\to \infty}\frac{\ln x^2}{\ln(x+4)^3}$

15. $\lim_{x\to -2}\frac{x+2}{x^2+2x-3}$

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Test Prep

16. If $f(x) = 2x^3 + 5$, then $\lim_{x \to 0} \frac{f(x) - f(0)}{x^3}$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) The limit does not exist.

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17. Functions $f, g,$ and h are twice-differentiable functions with $g(3) = h(3) = 5$. The line $y = 5 + \frac{1}{2}(x - 3)$ is tangent to both the graph of g at $x = 3$ and the graph of h at $x = 3$.

a. Find $h'(3)$.

b. Let a be the function given by $a(x) = 2x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(3)$.

c. The function h satisfies $h(x) = \frac{x^2 - 9}{1 - (f(x))^3}$ for $x \neq 3$. It is known that $\lim_{x \to 3} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 3} h(x) = 5$ to find $f(3)$ and $f'(3)$. Show the work that leads to your answers.