4.7 L'Hospital's Rule

Calculus

Solutions **Practice**

Find the following. Use L'Hôpital's when possible.

1. $\lim_{x\to1}\frac{x-1}{x^2-3x+2}=\frac{0}{0}$

$\lim_{x\to1}\frac{1}{2x-3}$

$\frac{1}{2-3}=\boxed{-1}$

2. $\lim_{x\to-5}\frac{x^2-2x-35}{x+5}=\frac{25+10-35}{0}=\frac{0}{0}$

$\lim_{x\to-5}\frac{2x-2}{1}$

$-10-2=\boxed{-12}$

3. $\lim_{x\to0}\frac{4x}{\ln(x+1)}=\frac{0}{0}$

$\lim_{x\to0}\frac{4}{\frac{1}{x+1}}$

$\frac{4}{1}=\boxed{4}$

4. $\lim_{x\to0}\frac{x-1}{x^2-3x+2}$

Direct substitution!

$\boxed{-\frac{1}{2}}$

5. $\lim_{x\to1}\frac{2(x^2-1)}{\ln x^2}=\frac{0}{0}$

$\lim_{x\to1}\frac{4x}{\frac{2}{x}}$

$\frac{4}{2}=\boxed{2}$

6. $\frac{d}{dx}\frac{6x^2+x}{\sin(x)}$ Quotient!

$\boxed{\frac{(12x+1)(\sin x)-(6x^2+x)\cos x}{\sin^2 x}}$

7. $\lim_{x\to0}\frac{2x^2}{e^x-1-x}=\frac{0}{0}$

$\lim_{x\to0}\frac{4x}{e^x-1}=\frac{0}{0}$

$\lim_{x\to0}\frac{4}{e^x}$

$\frac{4}{1}=\boxed{4}$

8. $\lim_{x\to0}\frac{2x^2}{1-\cos(4x)}=\frac{0}{0}$

$\lim_{x\to0}\frac{4x}{4\sin(4x)}=\frac{0}{0}$

$\lim_{x\to0}\frac{4}{16\cos(4x)}=\boxed{\frac{1}{4}}$

9. $\lim_{x\to0}\frac{\sqrt{4+x}-2}{x}=\frac{0}{0}$

$\lim_{x\to0}\frac{\frac{1}{2\sqrt{4+x}}}{1}$

$\frac{1}{2\sqrt{4}}=\boxed{\frac{1}{4}}$

10. $\lim_{x\to-3}\frac{x-1}{x^2+7x+10}$

Direct substitution

$\frac{-3-1}{9-21+10}=\frac{-4}{-2}$

$\boxed{2}$

11. $\lim_{x\to\infty}\frac{e^{2x}}{2x^2}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{2e^{2x}}{4x}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{4e^{2x}}{4}=\boxed{\infty}$

12. $\lim_{x\to0}\frac{\cos x-1}{x^2}=\frac{0}{0}$

$\lim_{x\to0}\frac{-\sin x}{2x}=\frac{0}{0}$

$\lim_{x\to0}\frac{-\cos x}{2}=\boxed{-\frac{1}{2}}$

13. $\frac{d}{dx}\frac{6x^2+x}{x+1}$ Quotient!

$\frac{(12x+1)(x+1)-(6x^2+x)(1)}{(x+1)^2}$

$\boxed{\frac{6x^2+12x+1}{(x+1)^2}}$

14. $\lim_{x\to\infty}\frac{\ln x^2}{\ln(x+4)^3}$ Use prop. of Logs

$\lim_{x\to\infty}\frac{2\ln x}{3\ln(x+4)}$

$\boxed{\frac{2}{3}}$

15. $\lim_{x\to-2}\frac{x+2}{x^2+2x-3}$ Direct Subst.

$\frac{-2+2}{4-4-3}=\frac{0}{-3}$

$\boxed{0}$

16. If $f(x) = 2x^3 + 5$, then $\lim_{x\to 0} \frac{f(x)-f(0)}{x^3}$ is $= \frac{0}{0}$

$\lim_{x\to 0} \frac{f'(x)}{3x^2} = \lim_{x\to 0} \frac{6x^2}{3x^2} = 2$

(A) 0 (B) 1 (C) 2 (D) 3 (E) The limit does not exist.

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17. Functions $f, g,$ and h are twice-differentiable functions with $g(3) = h(3) = 5$. The line $y = 5 + \frac{1}{2}(x-3)$ is tangent to both the graph of g at $x = 3$ and the graph of h at $x = 3$.

a. Find $h'(3)$.

Slope of h at $x=3$ is $\frac{1}{2}$

b. Let a be the function given by $a(x) = 2x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(3)$.

$a'(x) = 6x^2 h(x) + 2x^3 h'(x)$
$a'(3) = 6(9)h(3) + (54)h'(3)$
$54(5) + 54(\frac{1}{2}) = \boxed{297}$

c. The function h satisfies $h(x) = \frac{x^2-9}{1-(f(x))^3}$ for $x \neq 3$. It is known that $\lim_{x\to 3} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x\to 3} h(x) = 5$ to find $f(3)$ and $f'(3)$. Show the work that leads to your answers.

$1-(f(3))^3 = 0$
$[f(3)]^3 = 1$
$\boxed{f(3) = 1}$

$\lim_{x\to 3} \frac{2x}{-3[f(x)]^2 \cdot f'(x)} = 5$

$\frac{2(3)}{-3[1]^2 \cdot f'(3)} = 5$

$\frac{2}{-f'(3)} = 5$

$\boxed{-\frac{2}{5} = f'(3)}$