5.10 Introduction to Optimization

Calculus Name: _____ **CA #1**

Write out the equation that needs to be "optimized." This equation should be in one variable. You do NOT need to solve the problem. We will solve in the next lesson.

1. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

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2. An open rectangular box is to be made from a 9×12 inch piece of tin by cutting squares of side x inches from the corners and folding up the sides. What should x be to maximize the volume of the box?

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3. Find the point on the graph of the function $f(x) = x^2$ that is closest to the point $\left(2, \frac{1}{2}\right)$.

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4. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 20 - x^2$. Find the dimensions of the rectangle with the largest area.

5. A 216 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence right down the middle (parallel to one of the sides). What dimensions for the outer rectangle will require the smallest total length of fence?

6. Two towers are 45 feet apart. One is 15 feet high and the other is 20 feet high. There is a stake in the ground between the towers. The top of each tower has a wire tied to it that connects to the stake on the ground. Where should the stake be placed to use the least amount of wire?

7. A swimmer is 500 meters from the closest point on a straight shoreline. She needs to reach her house located 2000 meters down shore from the closest point. If she swims at $\frac{1}{2}$ m/s and she runs at 4 m/s, how far from her house should she come ashore so as to arrive at her house in the shortest time? *Hint*: $\text{time} = \frac{\text{distance}}{\text{rate}}$

Answers to 5.10 CA #1

1. $M = x + \frac{576}{x}$ or $M = \frac{192}{y} + 3y$	2. $V = 4x^3 - 42x^2 + 108x$	3. $d = \sqrt{(x-2)^2 + \left(x^2 - \frac{1}{2}\right)^2}$ or $d = \sqrt{(2-x)^2 + \left(\frac{1}{2} - x^2\right)^2}$	4. $A = 40x - 2x^3$
5. $P = \frac{648}{x} + 2x$ or $P = 3y + \frac{432}{y}$	6. $W = \sqrt{20^2 + (45-x)^2} + \sqrt{15^2 + x^2}$ or $W = \sqrt{20^2 + x^2} + \sqrt{15^2 + (45-x)^2}$	7. $T = 2\sqrt{500^2 + x^2} + \frac{(2000-x)}{4}$ or $T = 2\sqrt{500^2 + (2000-x)^2} + \frac{x}{4}$	