5.10 Introduction to Optimization

Notes

Write your questions and thoughts here!

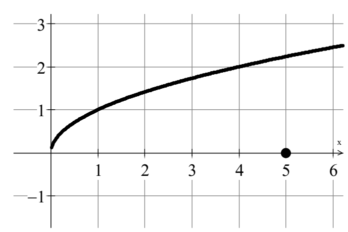
Optimize: To make the best or most effective use of a situation or resource.

Strategies for solving optimization problems:

1. Draw a picture (if applicable) and identify *known* and *unknown* quantities.
2. Write an equation (model) that will be optimized.
3. Write your equation in terms of a single variable.
4. Determine the desired max or min value with calculus techniques.
5. Determine the domain (endpoints) of your equation to verify if the endpoints represent a max or min.

1. Find two numbers whose sum is 30 and whose product is as large as possible.

2. What point on the graph $y = \sqrt{x}$ is closest to $(5,0)$.



3. Mr. Kelly needs to make an open-top box to store all his Magic cards. He has a piece of cardboard that is 14 by 30 inches. To do this, Mr. Kelly cuts out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

4. Two towers are 30 feet apart. One is 12 feet high and the other is 28 feet high. There is a stake in the ground between the towers. The top of each tower has a wire tied to it that connects to the stake on the ground. Where should the stake be placed to use the least amount of wire?

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Calculus

Practice

Write out the equation that needs to be "optimized." This equation should be in one variable. You do NOT need to solve the problem. We will solve in the next lesson.

1. What is the smallest product of two numbers given that one number is exactly 7 greater than the other number?

2. If the product of two positive numbers is 36, and the sum of the first number plus 4 times the second number is a minimum, what are the two numbers?

3. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

4. Sullivan and Brust are watching fireworks on the 4^{th} of July. They build towers to get a better view, and decide to work together by securing them to the same stake in the ground. They place the towers 40 feet apart. Mr. Brust's tower is 10 feet high and Mr. Sullivan's tower is 22 feet high. Where should the stake be placed to use the least amount of wire?

5. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?

6. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

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7. You are creating an open-top box with a piece of cardboard that is 16 X 30 inches. What size of square should be cut out of each corner to create a box with the largest volume?

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8. A rectangular pig pen using 300 feet of fencing is built next to an existing wall, so only three sides of fencing are needed. What dimensions should the farmer use to construct the pen with the largest possible area?

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9. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material? Volume of a cylinder is $V = \pi r^2 h$. Surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$

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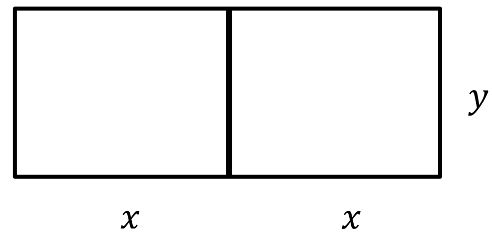
10. A power station is on one side of a river that is ½ mile wide, and a factory is 6 miles downstream on the other side. It costs $60,000 per mile to run power lines over land and $85,000 per mile to run them underwater. Find the most economical path for the transmission line from the power station to the factory. *Hint*: Total Cost = (on land cost)(distance on land) + (underwater cost)(distance in water)

11. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



Find the *largest* possible area of each object, given its boundaries. Draw a picture to represent each problem. You don't need to solve this problem, but set up the equation that you need to "optimize".

12. A rectangle is formed in Quadrant I with one corner at the origin and the other corner on the line $y = 8 - 2x$.

13. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 6 - x^2$.