5.11 Solving Optimization Problems

Calculus Name: ______________________________ **CA #2**

1. Calculator active problem. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 4 - 5\tan^{-1}(x - 7)$. Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

2. A particle is traveling along the x-axis and it's position from the origin can be modeled by $x(t) = t^3 + 3t^2 - 45t + 5$ where x is centimeters and t is minutes.
   a. On the interval $0 \leq t \leq 6$, find when the particle is farthest to the left.

   b. On the same interval, what is the particle's maximum speed?

3. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 6 - x^2$. What dimensions would give this rectangle the largest area?

4. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?

5. A rectangular pig pen using 300 feet of fencing is built next to an existing wall, so only three sides of fencing are needed. What dimensions should the farmer use to construct the pen with the largest possible area?

6. Two towers are 45 feet apart. One is 15 feet high and the other is 20 feet high. There is a stake in the ground between the towers. The top of each tower has a wire tied to it that connects to the stake on the ground. Where should the stake be placed to use the least amount of wire?

Answers to 5.11 CA #2

1. At $t = 8.0296$ because $v(t)$ changes sign from positive to negative.	2a. $t = 3$ 2b. 99 cm per minute	3. Optimize the equation: $A = 12x - 2x^3$ Answer: $2\sqrt{2} \times 4$
4. Optimize the equation: $d = \sqrt{x^2 + (2 - x^2)^2}$ Answer: $\left(-\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$ and $\left(\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$	5. Optimize the equation: $A = 150x - \frac{x^2}{2}$ or $A = 300x - 2x^2$ Answer: 75×150 feet	6. Optimize the equation: $A = \frac{x - 45}{\sqrt{(45 - x)^2 + 20^2}} + \frac{x}{\sqrt{x^2 + 15^2}}$ Answer: 19.2857 ft from the 15-ft tower