Calculus

Write your questions and thoughts here!

5.12 Behaviors of Implicit Relations

Notes

Implicit relationships still follow the same rules as functions. If $\frac{dy}{dx} = 0$ or $\frac{dy}{dx}$ does not exist at a point, then that point is a critical point. If $\frac{d^2y}{dx^2} > 0$ at a point, then the graph if concave up at that point.

1. Consider the curve $3x^3 + 3 = \ln(4y^2)$ in the xy-plane. At the point $\left(-1, \frac{1}{2}\right)$, is the curve increasing or decreasing?

2. Consider the curve $x^2 - 3 = e^y$ in the xy-plane. At the point $(-2, 0)$, is the curve concave up or concave down?

3. Consider the curve $y^3 - y = x^2$ in the xy-plane. It is known that $\frac{dy}{dx} = \frac{2x}{3y^2-1}$ and $\frac{d^2y}{dx^2} = \frac{2}{3y^2-1} - \frac{24x^2y}{(3y^2-1)^2}$. At the point $(0, 1)$ on the curve, is the point a relative max, relative min, or neither? Justify.

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Practice

Consider the curves in the xy-plane for each problem. At the point given point, is the curve increasing or decreasing? Justify your answer.

1. $x^2 - \frac{y^2}{2} = -1$ at $(-1, 2)$

2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$ at $(1, -8)$

3. $x^2 - 2xy + y^2 = 1$ at $(-1, -2)$

Consider the given differential equation $\frac{dy}{dx}$, where $y = f(x)$ is a particular solution with a given point. For each problem, determine if f has a relative minimum, a relative maximum, or neither at the given point. Justify your answer.

4. $\frac{dy}{dx} = y \sin x$ where $f(2\pi) = 1$

Instructions continued from last page.

5. $\frac{dy}{dx} = \frac{x}{y} + \ln x$ where $f(1) = -2$

6. $\frac{dy}{dx} = yx^2$ where $f(0) = -5$

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Test Prep

7. Consider the curve defined by $x^2 - y^2 - 5xy = 25$.
   a. Show that $\frac{dy}{dx} = \frac{2x-5y}{5x+2y}$

   b. Find the slope of the line tangent to the curve at each point on the curve when $x = 2$.

c. Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

d. Let x and y be functions of time t that are related by the equation $x^2 - y^2 - 5xy = 25$. At time $t = 3$, the value of x is 5, the value of y is 0, and the value of $\frac{dy}{dt}$ is -2. Find the value of $\frac{dx}{dt}$ at time $t = 3$.