5.2 Critical Points
Calculus

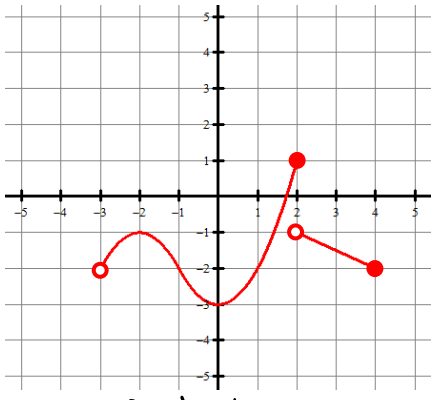
* included rel. extrema endpoints in case you included them

Solutions

Practice

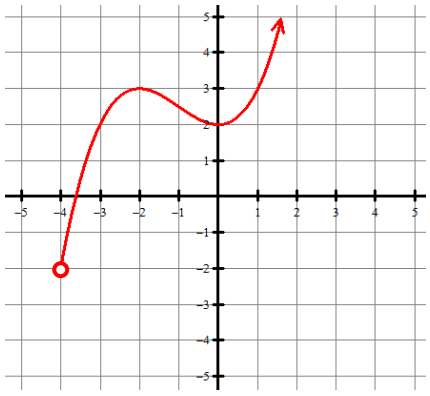
Find all extreme values. Identify the type and where they occur. For example, an answer could be written as "absolute max of 3 at $x = 1$."

1.



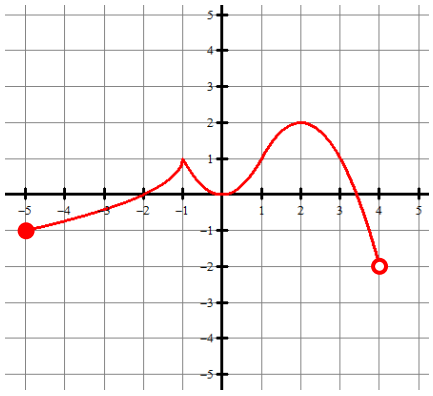
rel max of -1 at x=-2
abs min of -3 at x=0
abs max of 1 at x=2
*rel min of -2 at x=4

2.



rel max of 3 at x=-2
rel min of 2 at x=0

3.



rel max of 1 at x=-1
rel min of 0 at x=0
abs max of 2 at x=2
* rel min of -1 at x=-5

Find the critical points.

4. $f(x) = 4x^3 - 9x^2 - 12x + 3$

$f'(x) = 12x^2 - 18x - 12$

$6(2x^2 - 3x - 2) = 0$

$(2x+1)(x-2) = 0$

$x = -\frac{1}{2} \quad x = 2$

5. $g(t) = \frac{2}{t^2-4} = 2(t^2-4)^{-1}$

$g'(t) = -2(t^2-4)^{-2}(2t)$

$-\frac{4t}{(t^2-4)^2}$ $\quad 4t = 0$, $t = 0$

$t^2 - 4 \neq 0$

$t \neq \pm 2$ ← not in domain of $g(t)$.

$t = 0$

6. $h(x) = \sqrt[3]{x-2} = (x-2)^{\frac{1}{3}}$

$h'(x) = \frac{1}{3}(x-2)^{-\frac{2}{3}}$

$\frac{1}{3\sqrt[3]{(x-2)^2}}$

$x - 2 \neq 0$

$x \neq 2$

$x = 2$

7. $f(x) = (\ln x)^2$

$f'(x) = 2\ln x \cdot \frac{1}{x}$

$x \neq 0$ x=0 is not in the domain of f(x), so it can't be a critical pt.

$\ln x = 0$

$x = e^0 = 1$

$x = 1$

8. $h(x) = 2\sin\left(\frac{x}{2}\right)$ where $-2\pi \leq x \leq 2\pi$

$h'(x) = \cos\left(\frac{x}{2}\right)$

$\cos\left(\frac{x}{2}\right) = 0$

$\frac{x}{2} = -\frac{3\pi}{2}$ $\quad x = -3\pi$ ~~-3π~~

$\frac{x}{2} = -\frac{\pi}{2}$ $\quad x = -\pi$

$\frac{x}{2} = \frac{\pi}{2}$ $\quad x = \pi$

$\frac{x}{2} = \frac{3\pi}{2}$ $\quad x = 3\pi$ ~~3π~~

$x = \pm\pi$

9. $g(x) = e^x - x$

$g'(x) = e^x - 1$

$e^x - 1 = 0$

$e^x = 1$

$x = \ln(1)$

$x = 0$

5.2 Critical Points

Test Prep

10. **Calculator active problem.** The first derivative of the function f is given by $f'(x) = \frac{\sin^2 x}{x} - \frac{2}{9}$. How many critical values does f have on the open interval $(0, 10)$? Graph and count the number of zeros on $(0, 10)$

A) One (B) Two (C) Three (D) Four (E) Six

11. If f is a continuous, decreasing function on $[0,10]$ with a critical point at $(4, 2)$, which of the following statements must be false?

(A) $f(10)$ is an absolute minimum of f on $[0,10]$.

(B) $f(4)$ is neither a relative maximum nor a relative minimum.

(C) $f'(4)$ does not exist

(D) $f'(4) = 0$

(E) $f'(4) < 0$

The derivative must be zero or does not exist. It can't be negative if the point is a critical point.