5.3 Increasing and Decreasing Intervals

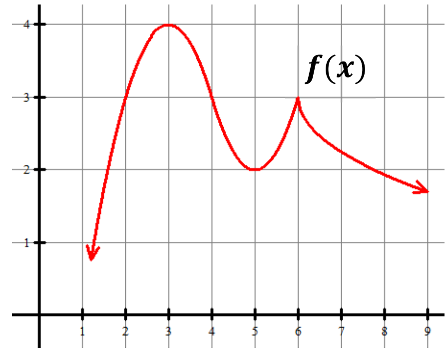
Notes

Write your questions and thoughts here!

Find the critical points of the graph of f.

When the slope of a function is **positive**, the function is **increasing**.

When the slope of a function is **negative**, the function is **decreasing**.



x							
Sign of $f'(x)$							

1. Find the intervals on which the function $f(x) = -x^2 - 4x - 1$ is increasing and decreasing and justify your answers.

   a. First find the critical points.

   b. In between the x-values, the derivative must be positive or negative.

   c. We can use a chart to help keep track of the information. Write the critical points of the derivative first.

x	
Sign of $f'(x)$	

   d. Answer statements with justification:

2. Find the intervals on which the function $f(x) = \frac{1}{3}x^3 - x^2 - 15x + 2$ is increasing and decreasing and justify your answers.

x	
Sign of $f'(x)$	

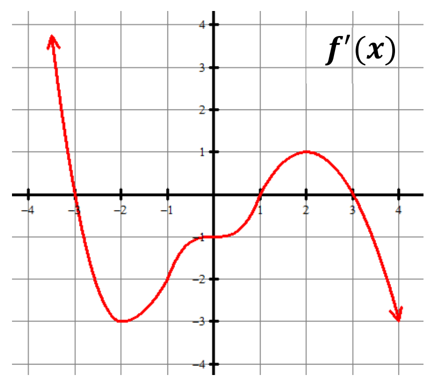
Answer statements with justification:

Graph of f'. Is f increasing or decreasing?

3. Determine the intervals where f is increasing and decreasing based on the graph of f'.

   Increasing:

   Decreasing:



Application of rate of change

If you want to know if something is increasing or decreasing, you look at the sign of its rate of change.

The sign of a rate of change can tell you if the variable is increasing or decreasing. Interpret the following:

$\frac{\text{students}}{\text{year}} > 0$	$\frac{\text{miles}}{\text{hour}} < 0$	$\frac{\text{mastery checks}}{\text{week}} > 0$	$\frac{\text{virus cases}}{\text{month}^2} < 0$

4. The rate of change of fruit flies in Mr. Kelly's kitchen at time t days is modeled by $R(t) = 2t\cos(t^2)$ flies per day. Show that the number of flies is decreasing at time $t = 3$.

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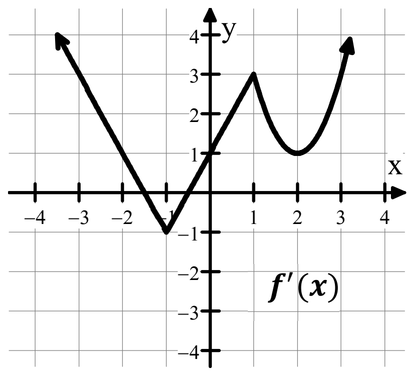
Calculus

Practice

The following graphs show the derivative of f, f'. Identify the intervals when f is increasing and decreasing. Include a justification statement.

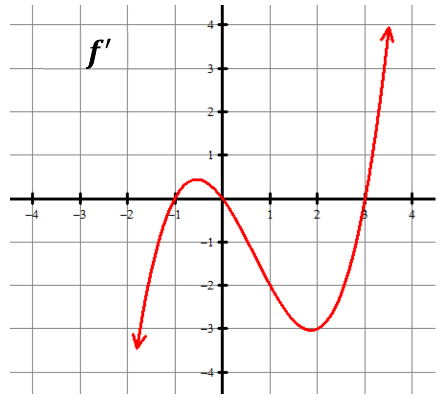
1.



Increasing:

Decreasing:

2.



Increasing:

Decreasing:

For each function, find the intervals where it is increasing and decreasing, and JUSTIFY your conclusion. Construct a sign chart to help you organize the information, but do not use a calculator.

3. $f(x) = x^3 - 12x + 1$

4. $g(x) = x^2(x - 3)$

5. $f(x) = x^2e^x$

6. $g(t) = 12(1 + \cos t)$ on the interval $(0, 2\pi)$

The derivative f' is given for each problem. Use a calculator to help you answer each question about f.		
7. $f'(x) = \frac{x+3e^{-x}}{x^2+0.8}$. On what intervals is f increasing?	8. $f'(x) = -\sin x - x\cos x$ for $0 \le x \le \pi$. On which interval(s) is f decreasing?	9. $f'(x) = \frac{1}{x} - e^x \sin x$ for $0 < x \le 4$. On what intervals is f decreasing?

For #10-12, calculator use is encouraged.

10. The rate of money brought in by a particular mutual fund is represented by $m(t) = \left(\frac{e}{2}\right)^t$ thousand dollars per year where t is measured in years. Is the amount of money from this mutual fund increasing or decreasing at time $t = 5$ years? Justify your answer.

11. The number of hair follicles on Mr. Sullivan's scalp is measured by the function $h(t) = 500e^{-t}$ where t is measured in years. Is the amount of hair increasing or decreasing at $t = 7$ years? Justify your answer.

12. The rate at which rainwater flows into a street gutter is modeled by the function $G(t) = 10\sin\left(\frac{t^2}{30}\right)$ cubic feet per hour where t is measured in hours and $0 \le t \le 8$. The gutter's drainage system allows water to flow out of the gutter at a rate modeled by $D(t) = -0.02x^3 + 0.05x^2 + 0.87x$ for $0 \le t \le 8$. Is the amount of water in the gutter increasing or decreasing at time $t = 4$ hours? Give a reason for your answer.

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Test Prep

13.

x	1	2	3	4	5
$f(x)$	-6	-1	3	6	8

The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

(A) f' is negative and decreasing for $1 \le x \le 5$.

(B) f' is negative and increasing for $1 \le x \le 5$.

(C) f' is positive and decreasing for $1 \le x \le 5$.

(D) f' is positive and increasing for $1 \le x \le 5$.

14. Let f be the function given by $f(x) = 4 - x$. g is a function with derivative given by

$$g'(x) = f(x)f'(x)(x-2)$$

On what intervals is g decreasing?

(A) $(-\infty, 2]$ and $[2, \infty)$

(B) $(-\infty, 2]$ only

(C) $[2, 4]$ only

(D) $[2, \infty)$ only

(E) $[4, \infty)$ only

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15. Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 2t^3 - 4t^2 + 4$.

(a) Is particle X moving toward the left or toward the right at time $t = 2$? Give a reason for your answer.

(b) At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

(c) A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 4t + 5$. At any time t, $t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a rectangle in the first quadrant. Find the rate of change of the area of the rectangle at time $t = 2$. Show the work that leads to your answer.