5.5 Determine Absolute Extrema from Candidates

Calculus

Practice

Find the absolute maximum value and the absolute minimum value of the function on the given interval. Remember to show that you checked ALL the candidates.

1. $f(x) = 1 + (x+1)^2, \quad [-2, 5]$

$f'(x) = 2(x+1)$

$2x + 2 = 0$

$x = -1$

$f(-2) = 2$

$f(-1) = \boxed{1} \leftarrow$ abs min

$f(5) = \boxed{37} \leftarrow$ abs max

2. $f(x) = 2x^3 + 3x^2 + 4 \quad [-2, 1]$

$f'(x) = 6x^2 + 6x$

$6x(x+1) = 0$

$x = 0 \quad x = -1$

$f(-2) = \boxed{0} \leftarrow$ abs min

$f(-1) = 5$

$f(0) = 4$

$f(1) = \boxed{9} \leftarrow$ abs max

3. $f(x) = \frac{x}{x^2+1}, \quad [-2, 2]$

$f'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2} = 0$

$x = \pm 1$

$f(-2) = -0.4$

$f(-1) = \boxed{-0.5} \leftarrow$ abs min

$f(1) = \boxed{0.5} \leftarrow$ abs max

$f(2) = 0.4$

4. $f(x) = \sin\left(x + \frac{\pi}{4}\right), \quad \left[0, \frac{7\pi}{4}\right]$

$f'(x) = \cos\left(x + \frac{\pi}{4}\right) = 0$

$x + \frac{\pi}{4} = \frac{\pi}{2} \quad x + \frac{\pi}{4} = \frac{3\pi}{2} \quad x + \frac{\pi}{4} = \frac{5\pi}{2}$

$x = \frac{\pi}{4} \quad x = \frac{5\pi}{4} \quad \cancel{x = \frac{9\pi}{4}}$

$f(0) = \frac{\sqrt{2}}{2}$

$f\left(\frac{\pi}{4}\right) = 1 \leftarrow$ abs max

$f\left(\frac{5\pi}{4}\right) = \boxed{-1} \leftarrow$ abs min

$f\left(\frac{7\pi}{4}\right) = 0$

5. $g(x) = xe^{2x} \quad [-1, 1]$

$g'(x) = (1)e^{2x} + xe^{2x} \cdot 2$

$e^{2x}(1 + 2x) = 0$

$x = -\frac{1}{2}$

$g(-1) = -e^{-2} = -\frac{1}{e^2}$

$g\left(-\frac{1}{2}\right) = -\frac{1}{2}e^{-1} = -\frac{1}{2e} \leftarrow$ abs min

$g(1) = e^2 \leftarrow$ abs max

6. $f(x) = x^3 + 2x^2 + x - 5 \quad [-2, 2]$

$f'(x) = 3x^2 + 4x + 1$

$(3x+1)(x+1) = 0$

$x = -\frac{1}{3} \quad x = -1$

$f(-2) = -7 \leftarrow$ abs min

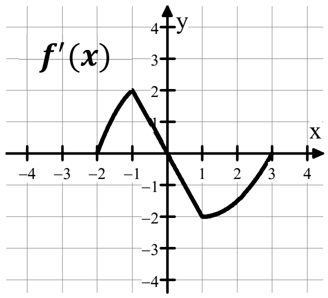
$f(-1) = -5$

$f\left(-\frac{1}{3}\right) = -5.148$

$f(2) = 13 \leftarrow$ abs max

The graph of f', the derivative of f, is shown for each problem. At what x-value does f have an absolute maximum and absolute minimum?

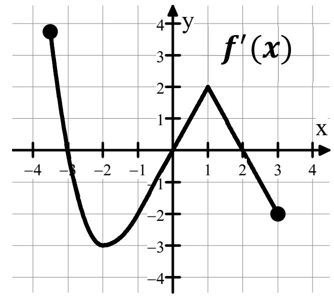
7. Domain: $-2 \le x \le 3$



Absolute **max** at $x =$ 0

Absolute **min** at $x =$ 3

8. Domain: $-3.5 \le x \le 3$



Absolute **max** at $x =$ −3

Absolute **min** at $x =$ 0

5.5 Determine Absolute Extrema from Candidates

Test Prep

9. **No calculator** allowed for this problem. Let f be the function defined by $f(x) = \cos^2 x - \cos x$ for $0 \le x \le \frac{3\pi}{2}$. Find the absolute maximum value and the absolute minimum value of f.

$f'(x) = 2\cos x(-\sin x) + \sin x$

$-2\cos x \sin x + \sin x = 0$

$\sin x(-2\cos x + 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$\sin x = 0$: $x = 0$, $x = \pi$

$\cos x = \frac{1}{2}$: $x = \frac{\pi}{3}$

$f(0) = 0$

$f(\frac{\pi}{3}) = -\frac{1}{4}$ ← abs min

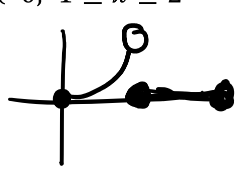
$f(\pi) = 2$ ← abs max

$f(\frac{3\pi}{2}) = 0$

10. Consider the function $f(x) = \begin{cases} x^2, & 0 \le x < 1 \\ 0, & 1 \le x \le 2 \end{cases}$. Which of the following is true?

$f' = \begin{cases} 2x \\ 0 \end{cases}$

$x = 0$



no max

$f(0) = 0$

$f(1) = 0$

$f(2) = 0$

all abs min

(A) f attains an absolute maximum value of 1.

(B) f attains an absolute minimum value of 0.

(C) f attains an absolute maximum value of 1 somewhere on the interval $[0, 2]$.

(D) f does not attain an absolute minimum value.

(E) Both (A) and (C).

11. A particle moves along the y-axis so that its velocity at time t, $0 \le t \le 6$, is given by $v(t) = 2(t-2)(t-5)$. Find the minimum velocity of the particle.

$v'(t) = 2(1)(t-5) + 2(t-2)(1)$

$v'(t) = 2t - 10 + 2t - 4$

$v'(t) = 4t - 14$

$4t - 14 = 0$

$t = 7/2$

$v(0) = 20$

$v(7/2) = -4.5$ minimum velocity

$v(6) = 8$

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12. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\cos t$ for $0 \le t \le 2\pi$. Find the time t at which the particle is farthest to the right.

$x'(t) = e^{-t}(-1)\cos t + e^{-t}(-\sin t)$

$-e^{-t}(\cos t + \sin t) = 0$

$-e^{-t} = 0$ or $\cos t + \sin t = 0$

not possible

$\cos t = -\sin t$

$t = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x(0) = 1$

$x(\frac{3\pi}{4}) \approx -0.067$

$x(\frac{7\pi}{4}) \approx 0.003$

$x(2\pi) \approx 0.002$

$\boxed{t = 0}$

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13. Find the maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = \frac{2}{3}t^3 - 4t^2 + 8t - 2$.

Find abs max of $a(t)$ on the interval!

$a(t) = 2t^2 - 8t + 8$

$a'(t) = 4t - 8$

Critical point: $4t - 8 = 0$

$t = 2$

$a(0) = \boxed{8}$ ← max

$a(2) = 0$

$a(3) = 2$