

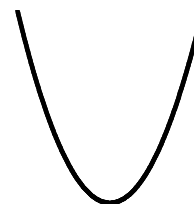
Write your questions
and thoughts here!

What is concavity? The state or quality of being concave.

Concave up:

Concave down:

If a function is concave up (like a parabola), what is f' is doing.



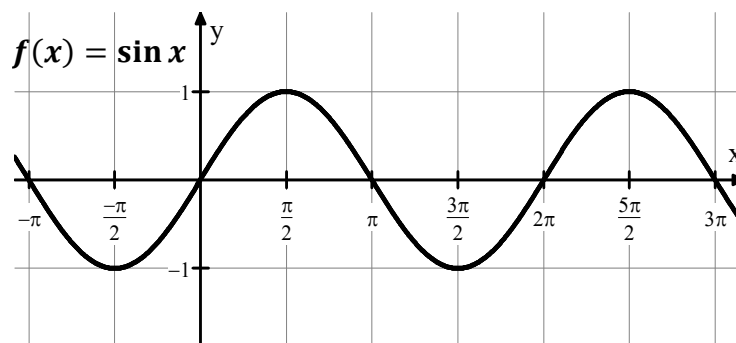
If f is concave up, then f' is

If f is concave down, then f'

This leads us to the following...

$f'' > 0$ means f is

$f'' < 0$ means f is



1. Find the intervals of concavity for $f(x) = \frac{1}{4}x^4 - 6x^2 + x - 3$.

Point of Inflection

There is a point of inflection of f at $x = c$ if $f(c)$ is defined and f'' changes signs at $x = c$.

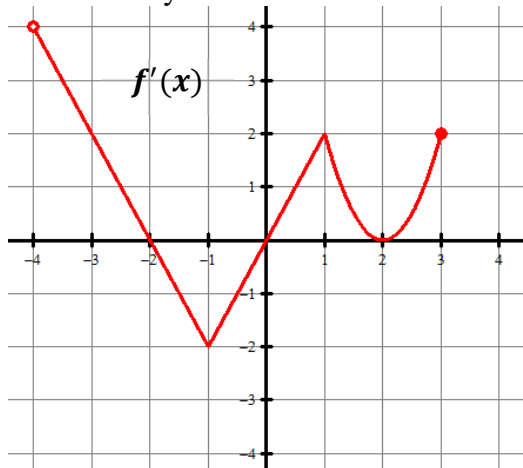
In other words, a point of inflection is where the graph

Write your questions and thoughts here!

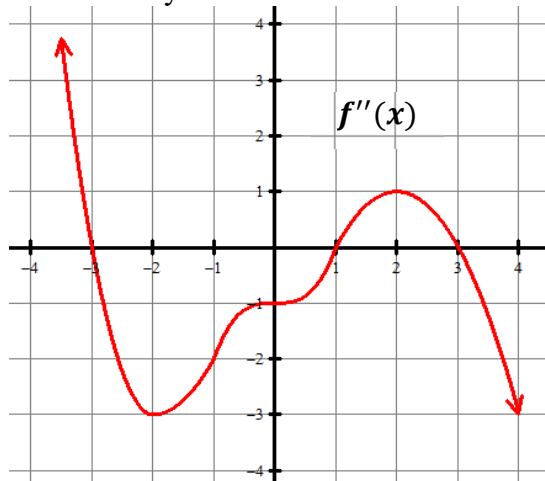
Two common **mistakes** when finding a point of inflection

1. Assuming that $f''(x) = 0$ means there is a point of inflection.
2. Assuming that $f''(x) < 0$ means there is no point of inflection.

2. Given the graph of $f'(x)$, find the points of inflection and state the intervals of concavity.



3. Given the graph of $f''(x)$, find the points of inflection and state the intervals of concavity.



4. Does the line tangent to the graph of $f(x) = xe^{-x}$ at $x = 1$ lie above or below the graph of f ? Why?

5.6 Determining Concavity

Calculus

Practice

1.

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	Positive	0	Negative

Use the table above to find the following.

Intervals where $g(x)$ is concave up:

Intervals where $g(x)$ is concave down:

Point(s) of Inflection:

Find the point(s) of inflection for each function. Justify your answer.

2. $f(x) = \sin \frac{x}{2}$ on the interval $(-\pi, 3\pi)$

3. $f(x) = e^{-x^2}$

4. $h(x) = (2x^2 - 5)^2$

5. $f(x) = 2x^4 - 8x + 3$

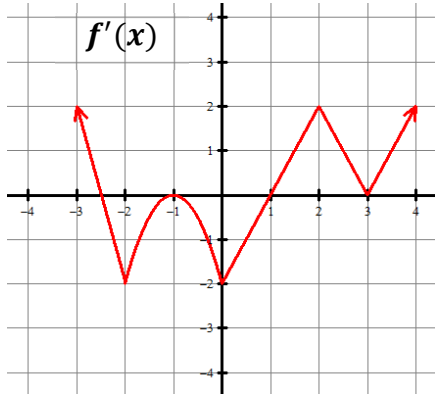
State the intervals of concavity and justify your answer.

6. $g(x) = \frac{x}{x-1}$

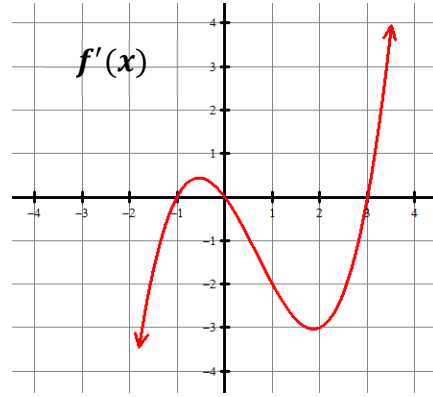
7. $f(x) = x^3 - 12x$

The graph of $f'(x)$ is shown. Find the point(s) of inflection.

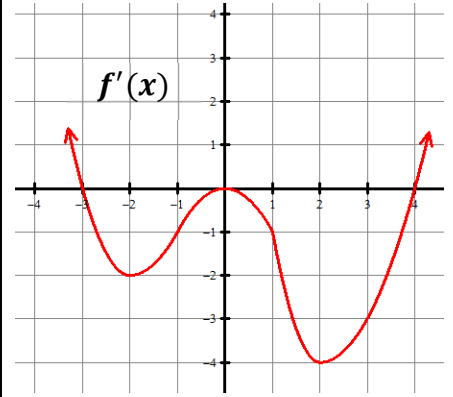
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9.

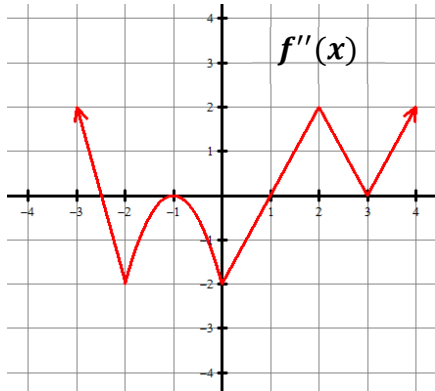


10.

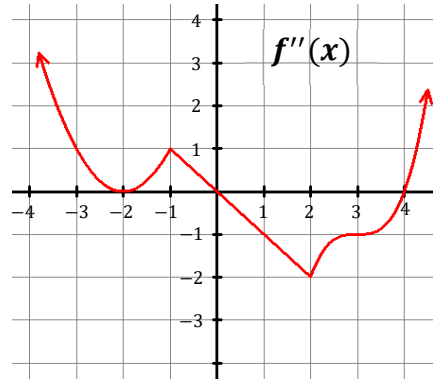


The graph of $f''(x)$ is shown. Find the point(s) of inflection.

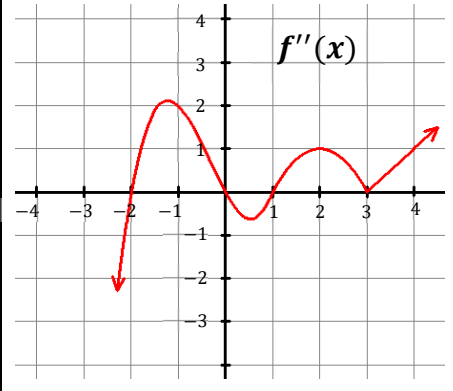
11.



12.



13.



Does the line tangent to the graph of h at the given value of x lie above or below the graph of h ? Why?

14. $h(x) = 2x^3 - 4x^2 - 3x$ at $x = -2$

15. $h'(x) = \frac{x^2 - 4}{x}$ at $x = 2$

5.6 Determining Concavity

16. **Calculator active problem.** Let $f''(x) = \sin x^2$. Which of the following three statements are true?

- I. f is concave up on $(0, 1.77)$ and $(2.51, 3.06)$.
- II. f is concave down on $(1.78, 2.50)$.
- III. f' is increasing on $(0, 1.77)$.

(A) I and II only

(D) II and III only

(B) I and III only

(E) III only

(C) I, II, and III

17. Consider the differential equation $\frac{dy}{dx} = 4x + y$. Find $\frac{d^2y}{dx^2}$. Determine the concavity of all solution curves for the given differential equation in Quadrant I. Give a reason for your answer.

18. Write an equation of the line tangent to $y = x^3 - 3x^2 - 4$ at its point of inflection.

19. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?

(A) -3

(B) 0

(C) 1

(D) 3

(E) It cannot be determined from the information given.