 Solutions **Practice**

1.

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	**Positive**	**0**	**Negative**

Use the table above to find the following.

Intervals where $g(x)$ is concave up: $(-3, -\frac{1}{2})$

Intervals where $g(x)$ is concave down: $(-\frac{1}{2}, 3)$

Point(s) of Inflection: $x = -\frac{1}{2}$

Find the point(s) of inflection for each function. Justify your answer.

2. $f(x) = \sin\frac{x}{2}$ on the interval $(-\pi, 3\pi)$

$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$f''(x) = -\frac{1}{4}\sin\left(\frac{x}{2}\right) = 0$

$\frac{x}{2} = -\pi \quad \frac{x}{2} = 0 \quad \frac{x}{2} = \pi$

$\cancel{x = -2\pi} \quad x = 0 \quad x = 2\pi$

x	$(-\pi, 0)$	0	$(0, 2\pi)$	2π	$(2\pi, 3\pi)$
$f''(x)$	+	0	−	0	+

Points of inflection at $x=0$ and $x=2\pi$ b/c f'' changes sign.

3. $f(x) = e^{-x^2}$

$f'(x) = e^{-x^2}(-2x)$

$f''(x) = e^{-x^2}(-2x)(-2x) + e^{-x^2}(-2)$

$e^{-x^2}(4x^2 - 2) = 0$

$x = \pm\sqrt{\frac{1}{2}}$

x	$(-\infty, -\sqrt{\frac{1}{2}})$	$-\sqrt{\frac{1}{2}}$	$(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}})$	$\sqrt{\frac{1}{2}}$	$(\sqrt{\frac{1}{2}}, \infty)$
$f''(x)$	+	0	−	0	+

Pts. of inflection at $x = \pm\sqrt{\frac{1}{2}}$ b/c f'' changes sign.

4. $h(x) = (2x^2 - 5)^2$

$h'(x) = 2(2x^2 - 5)(4x) = 16x^3 - 40x$

$h''(x) = 48x^2 - 40 = 0$

$x^2 = \frac{5}{6}$

$x = \pm\sqrt{\frac{5}{6}}$

x	$(-\infty, -\sqrt{\frac{5}{6}})$	$-\sqrt{\frac{5}{6}}$	$(-\sqrt{\frac{5}{6}}, \sqrt{\frac{5}{6}})$	$\sqrt{\frac{5}{6}}$	$(\sqrt{\frac{5}{6}}, \infty)$
$f''(x)$	+	0	−	0	+

Pts. of inflection at $x = \pm\sqrt{\frac{5}{6}}$ because f'' changes sign.

5. $f(x) = 2x^4 - 8x + 3$

$f'(x) = 8x^3 - 8$

$f''(x) = 24x^2 = 0$

$x = 0$

x	$(-\infty, 0)$	0	$(0, \infty)$
$f''(x)$	+	0	+

No pt. of inflection because $f''(x)$ does not change signs.

State the intervals of concavity:

6. $g(x) = \frac{x}{x-1}$

$g'(x) = \frac{(1)(x-1) - x(1)}{(x-1)^2}$

$g'(x) = \frac{-1}{(x-1)^2}$

$g'(x) = -(x-1)^{-2}$

$g''(x) = 2(x-1)^{-3}$

$g''(x) = \frac{2}{(x-1)^3}$

g'' DNE at $x=1$, but it is not a pt. of inflection b/c $g(1)$ DNE.

x	$(-\infty, 1)$	1	$(1, \infty)$
g''	−		+

concave down on $(-\infty, 1)$ concave up on $(1, \infty)$

7. $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

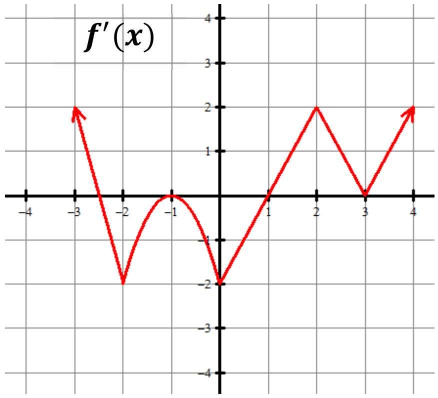
$x = 0$

x	$(-\infty, 0)$	0	$(0, \infty)$
$f''(x)$	−	0	+

Concave down on $(-\infty, 0)$ b/c $f'' < 0$.
Concave up on $(0, \infty)$ b/c $f'' > 0$.

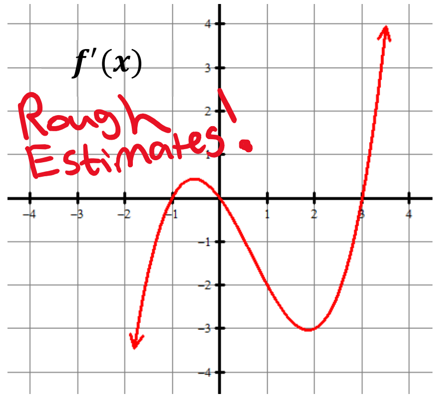
The graph of $f'(x)$ is shown. Find the point(s) of inflection.

8.



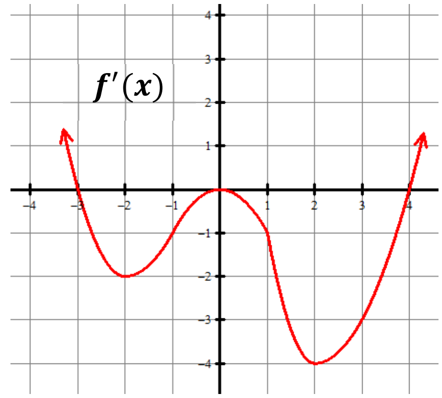
Pt of Inf: $x = -2, -1, 0$
$2, 3$

9.



Pts of Inf: $x = -0.5$
$x = 2$

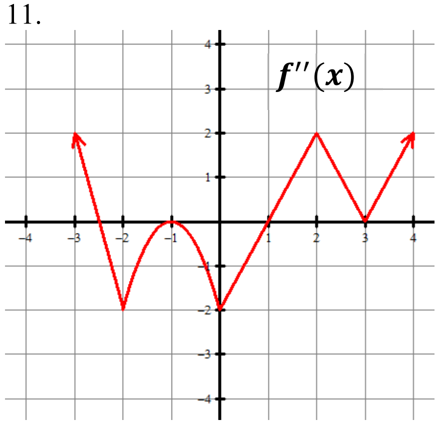
10.



Pts of Inf: $= -2$
$x = 0$
$x = 2$

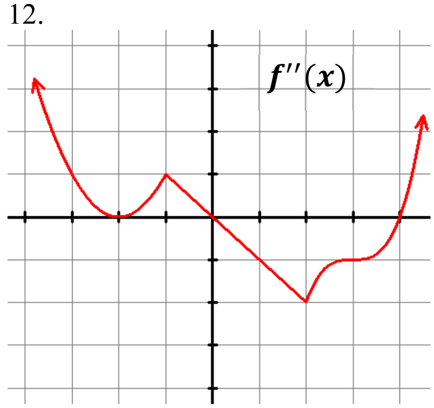
The graph of $f''(x)$ is shown. Find the point(s) of inflection.

11.



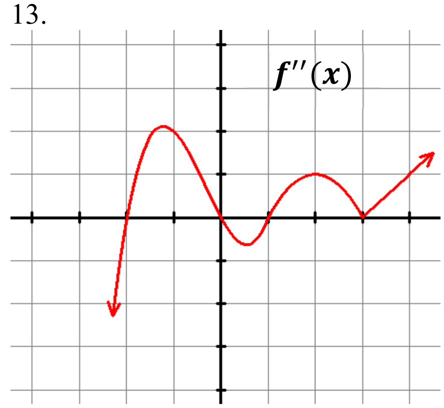
pts of inf: $x = -2.5$
$x = 1$

12.



pts of inf: $x = 0$
$x = 4$

13.



Pts. of inf: $x = -2$
$x = 0$
$x = 1$

Does the line tangent to the graph of h at the given value of x lie above or below the graph of h? Why?

14. $h(x) = 2x^3 - 4x^2 - 3x$ at $x = -2$

$h'(x) = 6x^2 - 8x - 3$

$h''(x) = 12x - 8$

$h''(-2) < 0$

$\boxed{\text{Above b/c } h''(-2) < 0.}$

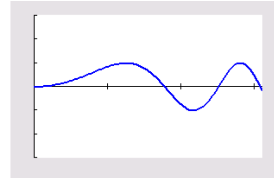
15. $h'(x) = \frac{x^2-4}{x}$ at $x = 2$

$h''(x) = \frac{(2x)(x) - (x^2-4)(1)}{x^2} = \frac{x^2+4}{x^2}$

$h''(2) > 0$

$\boxed{\text{Below b/c } h''(2) > 0.}$

16. **Calculator active problem.** Let $f''(x) = \sin x^2$. Which of the following three statements are true?



✓ I. f is concave up on $(0, 1.77)$ and $(2.51, 3.06)$.
✓ II. f is concave down on $(1.78, 2.50)$.
✓ III. f' is increasing on $(0, 1.77)$. $= f'' > 0$

(A) I and II only
(B) I and III only
(C) I, II, and III
(D) II and III only
(E) III only

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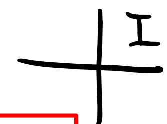
17. Consider the differential equation $\frac{dy}{dx} = 4x + y$. Find $\frac{d^2y}{dx^2}$. Determine the concavity of all solution curves for the given differential equation in Quadrant I. Give a reason for your answer.

both x and y are positive



$$\frac{d^2y}{dx^2} = 4 + \frac{dy}{dx}$$
$$= 4 + (4x + y)$$
$$\frac{d^2y}{dx^2} = 4 + 4x + y$$

$\boxed{\text{Concave up because } \frac{d^2y}{dx^2} > 0 \text{ if } x \text{ and } y \text{ are positive.}}$

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18. Write an equation of the line tangent to $y = x^3 - 3x^2 - 4$ at its point of inflection.

$y' = 3x^2 - 6x$
$y'' = 6x - 6$
$6x - 6 = 0$
$x = 1$

x	$(-\infty, 1)$	1	$(1, \infty)$
y''	$-$	0	$+$

$y(1) = (1)^3 - 3(1)^2 - 4 = -6$
$y'(1) = 3(1)^2 - 6(1) = -3$

$\boxed{y + 6 = -3(x - 1)}$

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19. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

$y' = 3x^2 + 2ax + b$
$y'' = 6x + 2a$
$6(1) - 2a = 0$
$a = -3$

$-6 = (1)^3 - 3(1)^2 + b(1) - 4$
$-6 = -6 + b$
$0 = b$

(A) -3
(B) 0
(C) 1
(D) 3
(E) It cannot be determined from the information given.