5.7 The Second Derivative Test

Calculus

Solutions

Practice

Find the relative extrema by using the Second Derivative Test. Justify your answer.

1. $f(x) = 5 + 3x^2 - x^3$

$f'(x) = 6x - 3x^2$

$3x(2-x) = 0$

$x = 0 \quad x = 2$

$f''(x) = 6 - 6x$

$f''(0) = 6$

$f''(2) = 6 - 6(2) = -6$

Rel. min at $x=0$ because $f'(0)=0$ and $f''(0)>0$.

Rel. max at $x=2$ because $f'(2)=0$ and $f''(2)<0$.

2. $h(x) = (2x-5)^2$

$h'(x) = 2(2x-5)\cdot(2)$

$8x - 20 = 0$

$x = 5/2$

$h''(x) = 8$

$h''(5/2) = 8$

Abs min at $x = 5/2$ because $h'(5/2)=0$ and $h''(5/2)>0$.

3. $g(x) = x + 2\sin x$ on the interval $(0, 2\pi)$

$g'(x) = 1 + 2\cos x = 0$

$\cos x = -1/2$

$x = 2\pi/3 \quad x = 4\pi/3$

$g''(x) = -2\sin x$

$g''(2\pi/3) = -2\left(\sqrt{3}/2\right) < 0$

$g''(4\pi/3) = -2\left(-\sqrt{3}/2\right) > 0$

Rel. max at $x = 2\pi/3$ b/c $g'(2\pi/3) = 0$ and $g''(2\pi/3) < 0$.

Rel. min at $x = 4\pi/3$ b/c $g'(4\pi/3) = 0$ and $g''(4\pi/3) > 0$.

4. $f(x) = 2x^4 - 8x + 3$

$f'(x) = 8x^3 - 8 = 0$

$x^3 = 1$

$x = 1$

$f''(x) = 24x^2$

$f''(1) = 24 > 0$

Abs. min at $x=1$ because $f'(1)=0$ and $f''(1)>0$.

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

$f'(x) = 4x^3 - 6x^2 \longrightarrow 2x^2(2x-3)=0$ $\quad f''(0)=0$

$f''(x) = 12x^2 - 12x$ $\quad x=0 \quad x=\frac{3}{2}$ $\quad f''(\frac{3}{2}) > 0$

$12x(x-1)=0$

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
$f''(x)$	+	0	−	0	+

2 pts of inflection

min.

(A) The graph of the function has two points of inflection, and the function has one relative extremum.

(B) The graph of the function has one point of inflection, and the function has two relative extrema.

(C) The graph of the function has two points of inflection, and the function has two relative extrema.

(D) The graph of the function has two points of inflection, and the function has three relative extrema.

(E) The function has no relative extremum.

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6. At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum?

$f'(x) = 4x^3 - 16x$ $\quad f''(x) = 12x^2 - 16$

$4x(x^2-4)=0$

$x=0, \; x=\pm 2$

$f''(-2) > 0$

$f''(0) < 0$

$f''(2) > 0$

(A) 0 and −2 only (B) 0 and 2 only (C) 0 only

(D) −2 and 2 only (E) −2, 0, and 2 only

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means y-value

7. What is the maximum value of the *derivative* of $f(x) = 3x^2 - x^3$?

$f'(x) = 6x - 3x^2 \leftarrow$ Find max of f'

$f'''(x) = -6 < 0$

$f''(x) = 6 - 6x$

f' is always concave down

$6 - 6x = 0$

$x = 1$

$f'(1) = 6 - 3 = 3$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4