6.10 Integrating with Long Division and Completing the Square

Calculus

Solutions

Practice

Find the indefinite integral.

1. $\int \frac{4x^2}{x-2}dx$

$$\begin{array}{r|rrr} 2 & 4 & 0 & 0 \\ & & 8 & 16 \\ \hline & 4 & 8 & 16 \end{array}$$

$$\int 4x + 8 + \frac{16}{x-2}\, dx$$

$$2x^2 + 8x + 16\ln|x-2| + C$$

2. $\int \frac{28x^2+33x-35}{4x+7}dx$

$$7x - 4 - \frac{7}{4x+7}$$

$$4x+7 \overline{)28x^2+33x-35}$$

$$-(28x^2+49x)$$

$$-16x-35$$

$$-(-16x-28)$$

$$-7$$

$u = 4x+7$

$\frac{du}{4} = dx$

$$\int 7x - 4 - \frac{7}{4x+7}\, dx$$

$$\frac{7}{2}x^2 - 4x - \int \frac{7}{u}\frac{du}{4}$$

$$\frac{7}{2}x^2 - 4x - \frac{7}{4}\ln|4x+7| + C$$

3. $\int \frac{1}{\sqrt{-x^2-4x-3}}dx$

$$-(x^2+4x+4) - 3 + 4$$

$$\int \frac{1}{\sqrt{1-(x+2)^2}}\, dx$$

$$\sin^{-1}(x+2) + C$$

4. $\int \frac{8}{x^2-10x+26}dx$

$$(x^2-10x+25) + 26 - 26$$

$$\int \frac{8}{(x-5)^2+1}\, dx$$

$$8\tan^{-1}(x-5) + C$$

5. $\int \frac{6}{\sqrt{-x^2-8x-7}}dx$

$-(x^2+8x+16)-7+16$

$$\int \frac{6}{\sqrt{9-(x+4)^2}}dx \cdot \frac{\frac{1}{\sqrt{9}}}{\frac{1}{\sqrt{9}}}$$

$$\int \frac{2}{\sqrt{1-\left(\frac{x+4}{3}\right)^2}}dx$$

$u=\frac{x+4}{3}$

$3du=dx$

$$6\sin^{-1}\left(\frac{x+4}{3}\right)+C$$

6. $\int \frac{14x^2+11x-5}{2x+1}dx$

$$\begin{array}{r} 7x+2-\frac{7}{2x+1} \\ 2x+1 \overline{)\,14x^2+11x-5} \\ -(14x^2+7x) \\ \hline 4x-5 \\ -(4x+2) \\ \hline -7 \end{array}$$

$$\int 7x+2-\frac{7}{2x+1}$$

$u=2x+1$

$\frac{du}{2}=dx$

$$\frac{7}{2}x^2+2x-\frac{7}{2}\ln|2x+1|+C$$

7. $\int \frac{1}{x^2-12x+37}dx$

$(x^2-12x+36)+37-36$

$$\int \frac{1}{(x-6)^2+1}dx$$

$$\tan^{-1}(x-6)+C$$

8. $\int \frac{5x^3+12x^2-38x-54}{5x+7}dx$

$$\begin{array}{r} x^2+x-9+\frac{9}{5x+7} \\ 5x+7 \overline{)\,5x^3+12x^2-38x-54} \\ -(5x^3+7x^2) \\ \hline 5x^2-38x-54 \\ -(5x^2+7x) \\ \hline -45x-54 \\ -(-45x-63) \\ \hline 9 \end{array}$$

$$\int x^2+x-9+\frac{9}{5x+7}$$

$u=5x+7$

$\frac{du}{5}=dx$

$$\frac{x^3}{3}+\frac{x^2}{2}-9x+\frac{9}{5}\ln|5x+7|+C$$

9. $\int \frac{3}{x^2+14x+49}dx$

$(x^2+14x+49)+49-49$

$\int \frac{3}{(x+7)^2}dx$

$u=x+7$
$du=dx$

$\int 3u^{-2}\,du$

$\frac{3u^{-1}}{-1}+C = -\frac{3}{x+7}+C$

10. $\int \frac{x^3+2x^2+5}{x+2}dx$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 0 & 5 \\ & \downarrow & -2 & 0 & 0 \\ \hline & 1 & 0 & 0 & 5 \end{array}$$

$\int x^2+\frac{5}{x+2}dx$

$\frac{x^3}{3}+5\ln|x+2|+C$

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Test Prep

11. $\int \frac{16}{x^2-6x+25}dx$

$(x^2-6x+9)+25-9$

$= \int \frac{16}{(x-3)^2+16}dx \cdot \frac{\frac{1}{16}}{\frac{1}{16}} = \int \frac{1}{\frac{(x-3)^2}{16}+1}dx$

$\int \frac{1}{\left(\frac{x-3}{4}\right)^2+1}dx$

$u=\frac{1}{4}x-\frac{3}{4}$
$4du=dx$

(A) $16\ln|x^2-16x+25|+C$

(B) $\tan^{-1}\left(\frac{x-3}{4}\right)+C$

(C) $16\tan^{-1}(x-3)+C$

(D) $4\tan^{-1}\left(\frac{x-3}{4}\right)+C$

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12. $\int \frac{12}{\sqrt{-x^2-2x+3}}dx$

$-(x^2+2x+1)+3+1$

$= \int \frac{12}{\sqrt{4-(x+1)^2}}dx \cdot \frac{\frac{1}{\sqrt{4}}}{\frac{1}{\sqrt{4}}} = \int \frac{6}{\sqrt{1-\frac{(x+1)^2}{4}}}dx$

$\int \frac{6}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}dx$

$u=\frac{1}{2}x+\frac{1}{2}$
$2du=dx$

(A) $12\sin^{-1}\left(\frac{x+1}{2}\right)+C$

(B) $12\sin^{-1}\left(\frac{x-1}{2}\right)+C$

(C) $6\sin^{-1}\left(\frac{x-1}{2}\right)+C$

(D) $6\sin^{-1}\left(\frac{x+1}{2}\right)+C$