

6.12 Linear Partial Fractions

Calculus

Solutions

Practice

Evaluate using partial fractions

$$1. \int \frac{x-12}{x^2-4x} dx = \int \frac{x-12}{x(x-4)} dx$$

$$\frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$$

$$x-12 = A(x-4) + Bx$$

Let $x=4$



$$-8 = 4B$$

$$-2 = B$$

Let $x=0$



$$-12 = -4A$$

$$3 = A$$

$$\int \frac{3}{x} + \frac{-2}{x-4} dx =$$

$$3 \ln|x| - 2 \ln|x-4| + C$$

or

$$\ln \left| \frac{x^3}{(x-4)^2} \right| + C$$

$$2. \int \frac{2x}{x^2-4} dx = \int \frac{2x}{(x-2)(x+2)} dx$$

$$\frac{2x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$2x = A(x+2) + B(x-2)$$

Let $x=-2$



$$-4 = -4B$$

$$1 = B$$

Let $x=2$



$$4 = 4A$$

$$1 = A$$

$$\int \frac{1}{x-2} + \frac{1}{x+2}$$

$$\ln|x-2| + \ln|x+2|$$

$$\ln|x^2-4| + C$$

$$3. \int \frac{1}{(x+2)(x-3)(x+1)} dx$$

$$\frac{1}{(x+2)(x-3)(x+1)} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{x+1}$$

$$1 = A(x-3)(x+1) + B(x+2)(x+1) + C(x+2)(x-3)$$

Let $x=-2$



$$1 = A(-5)(-1)$$

$$A = \frac{1}{5}$$

Let $x=3$



$$1 = B(5)(4)$$

$$B = \frac{1}{20}$$

Let $x=-1$



$$1 = C(1)(-4)$$

$$C = -\frac{1}{4}$$

$$\int \frac{1}{5(x+2)} + \frac{1}{20(x-3)} - \frac{1}{4(x+1)}$$

$$\frac{1}{5} \ln|x+2| + \frac{1}{20} \ln|x-3| - \frac{1}{4} \ln|x+1| + C$$

$$4. \int \frac{x+2}{x^2+5x} dx$$

$$\frac{x+2}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5}$$

$$x+2 = A(x+5) + Bx$$

Let $x=0$



$$2 = A(5)$$

$$\frac{2}{5} = A$$

Let $x=-5$



$$-3 = B(-5)$$

$$\frac{3}{5} = B$$

$$\int \left(\frac{2}{5x} + \frac{3}{5(x+5)} \right) dx$$

$$\frac{2}{5} \ln|x| + \frac{3}{5} \ln|x+5| + C$$

$$5. \int \frac{2}{x(x-2)} dx$$

$$\frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$2 = A(x-2) + Bx$$

Let $x=0$



$$2 = A(-2)$$

$$-1 = A$$

Let $x=2$



$$2 = B(2)$$

$$1 = B$$

$$\int -\frac{1}{x} + \frac{1}{x-2} dx$$

$$-\ln|x| + \ln|x-2| + C$$

or

$$\ln\left|\frac{x-2}{x}\right| + C$$

7. For $0 < P < 50$, what is the antiderivative of

$$\frac{1}{P(50-P)} = \frac{A}{P} + \frac{B}{50-P}$$

$$1 = A(50-P) + BP$$

Let $P=0$



$$1 = 50A$$

$$\frac{1}{50} = A$$

Let $P=50$



$$1 = 50B$$

$$\frac{1}{50} = B$$

$$\int \frac{1}{50} \frac{1}{P} + \frac{1}{50} \frac{1}{50-P} dp$$

$$\frac{1}{50} \ln|P| - \frac{1}{50} \ln|50-P| + C$$

or

$$\ln\left|\left(\frac{P}{50-P}\right)^{\frac{1}{50}}\right| + C$$

$$6. \int \frac{x^3-11x-15}{x^2-2x-8} dx$$

* Long division first!

$$x^2-2x-8 \overline{) x^3-11x-15}$$

$$\begin{array}{r} x+2 + \frac{x+1}{x^2-2x-8} \\ -(x^3-2x^2-8x+16) \\ \hline 2x^2-3x-15 \\ -(-2x^2+4x-16) \\ \hline x+1 \end{array}$$

$$\int x+2 + \frac{x+1}{x^2-2x-8}$$

$$\frac{x+1}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$x+1 = A(x+2) + B(x-4)$$

Let $x=4$



$$5 = 6A$$

$$\frac{5}{6} = A$$

Let $x=-2$



$$-1 = -6B$$

$$\frac{1}{6} = B$$

$$\int x+2 + \frac{5}{6} \frac{1}{x-4} + \frac{1}{6} \frac{1}{x+2} dx$$

$$\frac{x^2}{2} + 2x + \frac{5}{6} \ln|x-4| + \frac{1}{6} \ln|x+2| + C$$

$$8. \int_0^1 \frac{1}{(x+5)(x+1)} dx$$

$$\frac{1}{(x+5)(x+1)} = \frac{A}{x+5} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x+5)$$

Let $x=-5$



$$1 = A(-4)$$

$$-\frac{1}{4} = A$$

Let $x=-1$



$$1 = B(4)$$

$$\frac{1}{4} = B$$

$$\int_0^1 \left(-\frac{1}{4} \frac{1}{x+5} + \frac{1}{4} \frac{1}{x+1}\right) dx$$

$$-\frac{1}{4} \ln|x+5| + \frac{1}{4} \ln|x+1| \Big|_0^1$$

$$\left[-\frac{1}{4} \ln(6) + \frac{1}{4} \ln(2)\right] - \left[-\frac{1}{4} \ln(5) + \frac{1}{4} \ln(1)\right]$$

$$\frac{1}{4} [-\ln 6 + \ln 2 + \ln 5]$$

$$\frac{1}{4} \left[\ln\left(\frac{10}{6}\right)\right] = \frac{1}{4} \ln\left(\frac{5}{3}\right)$$

$$9. \int_0^2 \frac{3}{(4x+1)(x+1)} dx = \frac{A}{4x+1} + \frac{B}{x+1}$$

$$3 = A(x+1) + B(4x+1)$$

$$\text{Let } x = -\frac{1}{4}$$



$$3 = A\left(\frac{3}{4}\right)$$

$$4 = A$$

$$\int_0^2 \frac{4}{4x+1} - \frac{1}{x+1} dx$$

u-sub!

$$\ln|4x+1| - \ln|x+1| \Big|_0^2$$

$$[\ln|9| - \ln|3|] - [\ln|1| - \ln|1|]$$

$$\ln 9 - \ln 3 - [0]$$

$$\ln\left(\frac{9}{3}\right)$$

$$\ln(3)$$

$$10. \int_2^3 \frac{3}{(x-1)(x+2)} dx = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3 = A(x+2) + B(x-1)$$

$$\text{Let } x = 1$$



$$3 = A(3)$$

$$1 = A$$

$$\int_2^3 \frac{1}{x-1} + \frac{-1}{x+2} dx$$

$$\ln|x-1| - \ln|x+2| \Big|_2^3$$

$$[\ln(2) - \ln(5)] - [\ln(1) - \ln(4)]$$

$$\ln\left(\frac{2}{5}\right) + \ln(4)$$

$$\ln\left(\frac{8}{5}\right)$$

No test prep for this lesson.