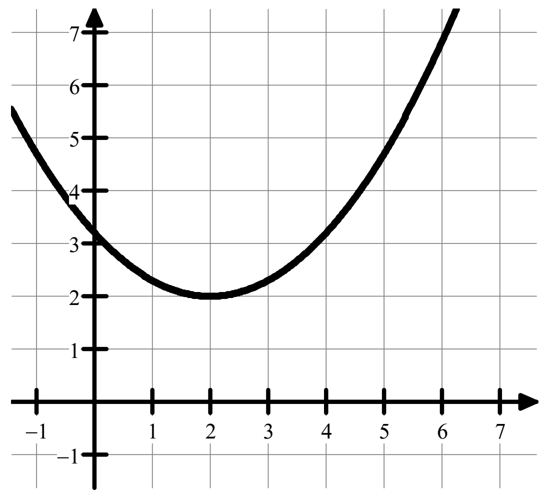
Calculus

6.2 Approximating Areas with Riemann Sums

Notes

Write your questions and thoughts here!

The graph of the function $g(x)$ is shown to the right. Approximate the area under the curve on the interval $[2, 6]$ with n subintervals by using a left-rectangular approximation method.

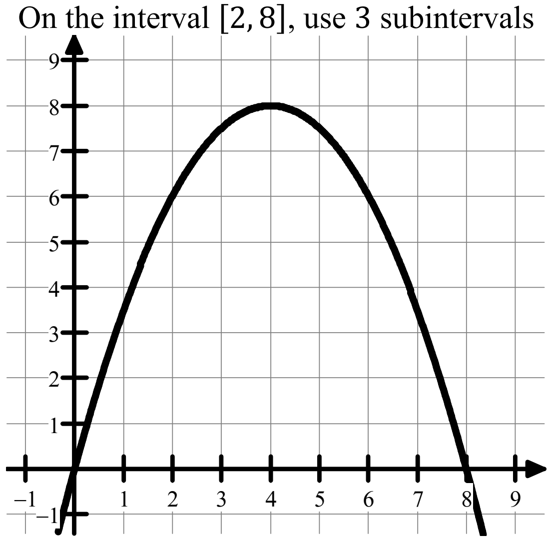


This approximation method is called a ____________________. It was named after a German mathematician named Bernhard Riemann.

Below is the graph of $f(x) = 4x - \frac{1}{2}x^2$. Use Riemann Sums to find the approximation of the area under the curve.

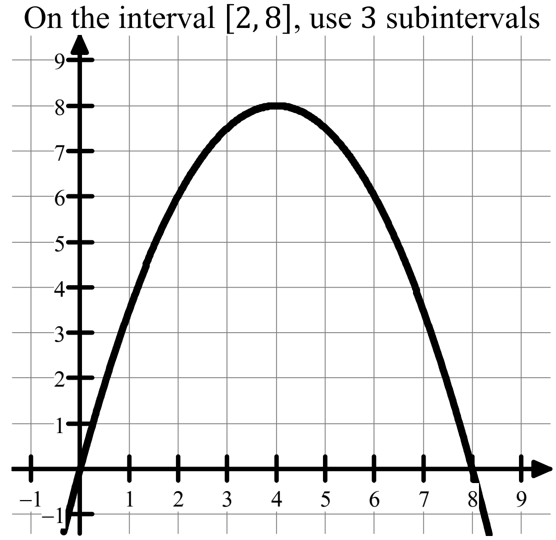
Left-Riemann Sum

On the interval $[2, 8]$, use 3 subintervals



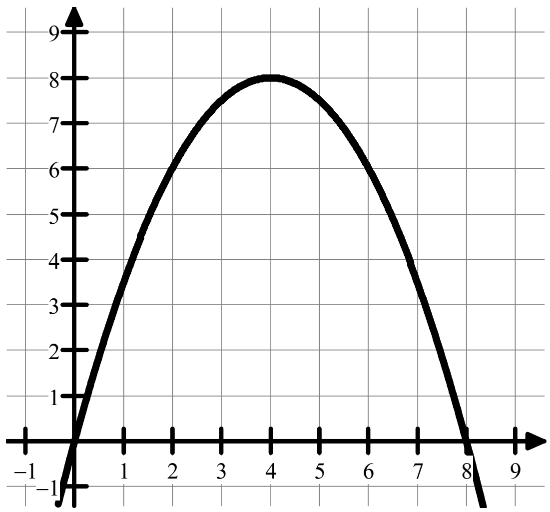
Right-Riemann Sum

On the interval $[2, 8]$, use 3 subintervals

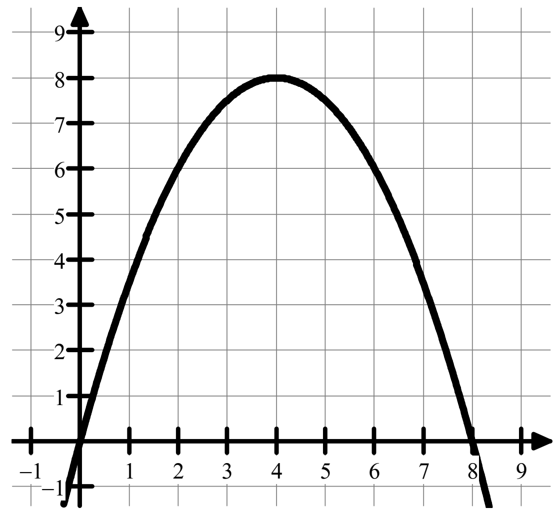


Write your questions and thoughts here!

Midpoint-Riemann Sum
On the interval $[2, 8]$, use 3 subintervals



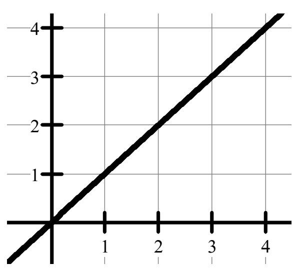
Trapezoidal Sum
On the interval $[2, 8]$, use 3 subintervals



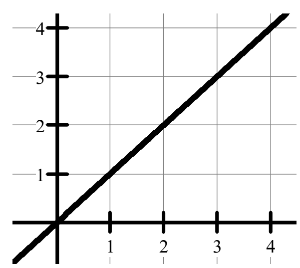
<u>Overestimate or Underestimate?</u>

Increasing function

Left-Riemann =

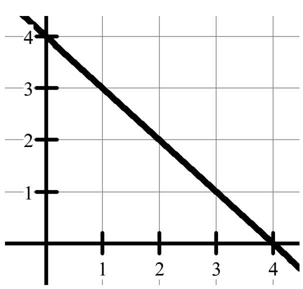


Right-Riemann =

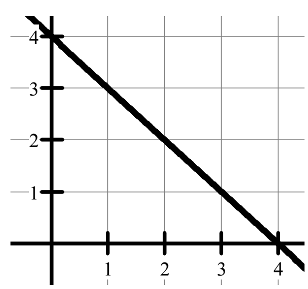


Decreasing function

Left-Riemann =



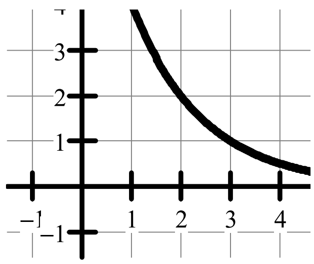
Right-Riemann =



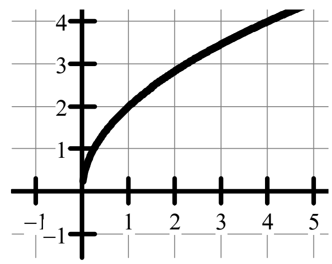
Write your questions and thoughts here!

Trapezoid estimation

Concave up =



Concave down =



Using Riemann Sums with a Table of Values

The rate at which water is being pumped into a tank is given by the continuous and increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 12$ minutes, is given below.

Time (minutes)	0	3	6	9	12
$\boldsymbol{R(t)}$ **(gallons/min)**	7	13	18	23	27

Use the following Riemann sums (with the given intervals), to estimate the number of gallons of water pumped into the tank during the 12 minutes.

Right-Riemann sum with 4 subintervals

Is the approximation greater or less than the true value? Why?

Left-Riemann sum with 4 subintervals

Is the approximation greater or less than the true value? Why?

Midpoint-Riemann sum with 2 subintervals

Trapezoidal sum with 4 subintervals

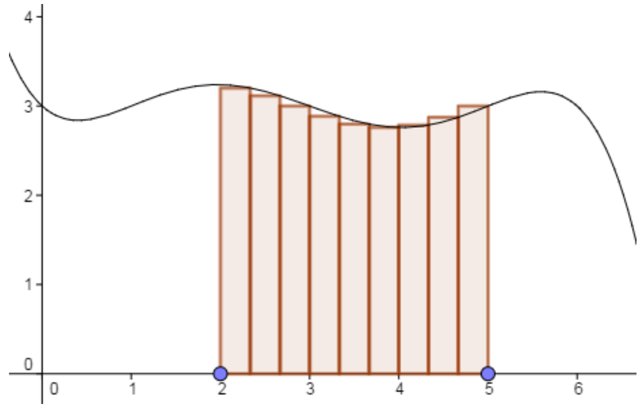
6.2 Approximating Areas with Riemann Sums

Calculus

Practice

1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

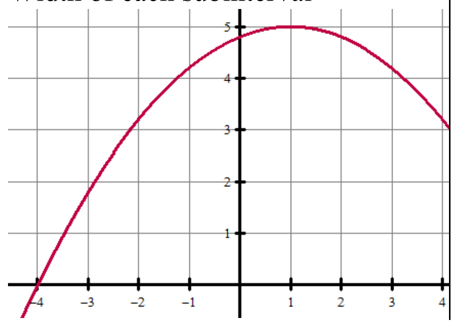
2. What is the width of each rectangle?



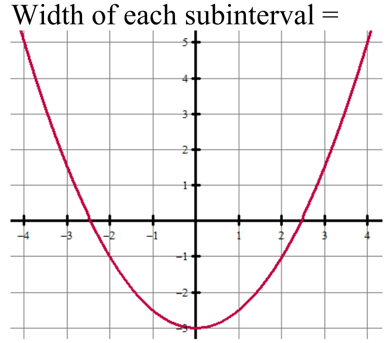
Sketch the following rectangular approximations. Find the width of each subinterval.

3. Midpoint on the interval [1,4] with $n = 6$ subintervals
Width of each subinterval =

4. Right Endpoint on [-2,2] with $n = 5$ subintervals
Width of each subinterval =

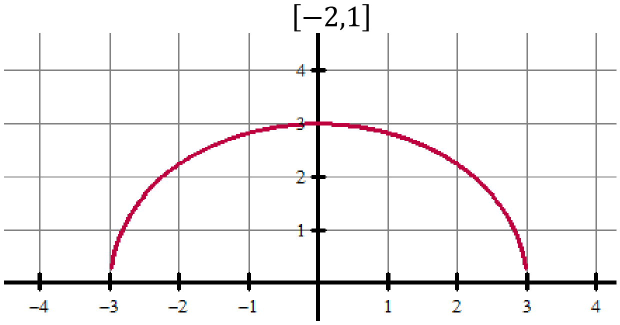


5. Left Endpoint on [-2,4] with $n = 10$ subintervals
Width of each subinterval =

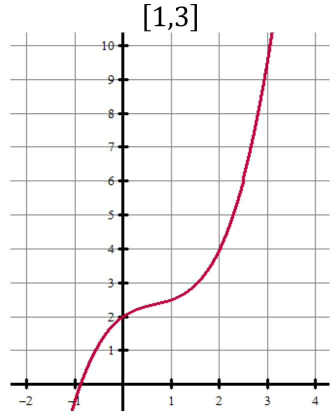


Approximate the area under the curve using the given Riemann Sum. Include a sketch!

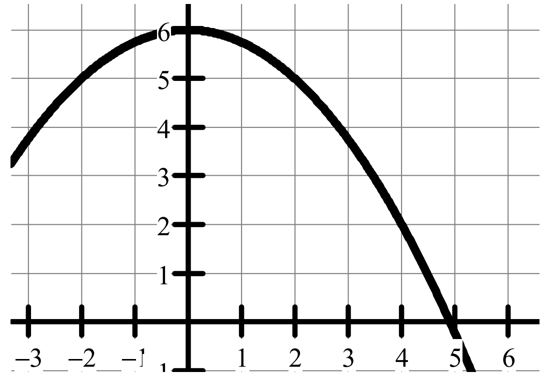
6. $f(x) = \sqrt{9 - x^2}$
Right Riemann sum with 3 subintervals on the interval $[-2,1]$



7. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$
Left Riemann sum with 4 subintervals on the interval $[1,3]$



8. $f(x) = 6 - \frac{1}{4}x^2$ Trapezoidal approximation with 3 subintervals on the interval $[-2, 4]$.



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Test Prep

9. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable increasing function of t. The table shows the population change in people per year recorded at selected times.

Time years	0	4	10	13	20
$y(t)$ people per year	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

b. What does your answer from part (a) represent?

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value? Why?

10. A rectangular pool gets deeper from one end of the pool to the other. The table shows the depth $h(x)$ of the water at 4-foot intervals from one end of the pool to the other.

position, x feet	0	4	8	12	16	20	24	28	32
$h(x)$ feet	6.5	8	9.5	10	11	11.5	12	13	13.5

a. Use the data from the table to find an approximation for $h'(10)$, and explain the meaning of $h'(10)$ in terms of the depth of the pool. Show the computations that lead to your answer.

b. Use a midpoint Riemann Sum with 4 subintervals to approximate the area under the curve from 0 to 32 feet.

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11. The rate at which customers are being served at StarBrusts is given by the continuous function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 10$ hours, is given below. At $t = 0$ there had already been 200 customers served.

Time hours	0	2	3	6	10
$R(t)$ people/hour	37	44	36	42	48

Use a trapezoidal sum with four subintervals to approximate how many customers had been served after 10 hours.