Calculus

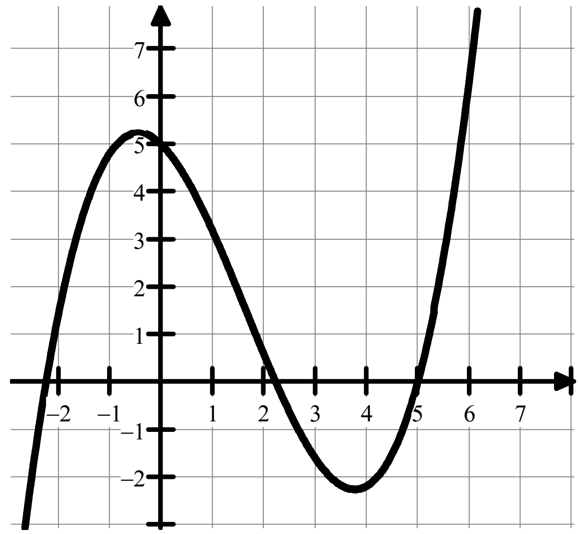
6.3 Summation Notation

Notes

Write your questions and thoughts here!

The graph of the function $f(x)$ is shown below. How can we approximate the area under the curve on the interval $[-1, 6]$?

Create a Reimann sum with n subintervals and have $n \to \infty$.



If n is the number of subintervals (rectangles) on the interval $[a, b]$, what is the width of each subinterval (rectangle)?

If $n \to \infty$ on the interval $[a, b]$, what does the width of each subinterval (rectangle) approach?

The sum of the area of all rectangles gives you the area under the curve.

We can represent this area by combining limits with summation notation.

Summation Notation

The area under the curve of $f(x)$ on the interval $[a, b]$ is represented by

$$\lim_{n\to\infty} \sum_{k=1}^{n}$$

where n represents the number of subintervals (rectangles) there are in the interval $[a, b]$ and k represents the kth subinterval.

Another way of writing the summation notation

Write your questions and thoughts here!

Definite Integral Notation

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{b-a}{n}\right)\cdot f\left(a+\frac{b-a}{n}k\right) =$$

The area under the curve of $f(x)$ on the interval $[a, b]$ is represented by .

Examples:

1. Rewrite the definite integral using summation notation.

$$\int_2^6 (x^2-3)\,dx =$$

2. Rewrite the summation notation expression as a definite integral.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{6}{n}\right)\left(4+\frac{6k}{n}\right)^2 =$$

a.	b.	c.

3. $\lim\limits_{n\to\infty}\frac{2}{n}\left(\frac{1}{\frac{2}{n}+3}+\frac{1}{\frac{4}{n}+3}+\frac{1}{\frac{6}{n}+3}+\cdots+\frac{1}{\frac{2n}{n}+3}\right)$

   Assuming the lower limit "a" is 0, write a definite integral that represents the above expression.

4. The expression $\frac{1}{10}\left(\cos\left(\frac{1}{10}\right)+\cos\left(\frac{2}{10}\right)+\cos\left(\frac{3}{10}\right)+\cdots+\cos\left(\frac{10}{10}\right)\right)$ is a Riemann sum approximation for what definite integral?

   Where is the 10? Why isn't it written in the integral?

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Practice

Write a definite integral that is equivalent to the given summation notation. The lower limit for the integral is also given to help you get started.

1. Integral's lower limit $= 0$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{n}\right)\left(\cos\left(\frac{\pi}{n}k\right)\right)$$

2. Integral's lower limit $= -3$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{5}{n}\right)\left(\sqrt[3]{-3+\frac{5k}{n}}\right)$$

3. Integral's lower limit $= 6$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{9}{n}\right)\left(\frac{1}{\left(6+\frac{9k}{n}\right)^2}\right)$$

Write a summation notation equivalent to the definite integral.

4. $\int_{-3}^{3} x^2\,dx$

5. $\int_{2}^{5} \frac{1}{x}\,dx$

6. $\int_{0}^{7} \sqrt{x}\,dx$

7. Which of the following expressions is equal to $\lim\limits_{n\to\infty}\frac{1}{n}\left(e^{1+\frac{1}{n}}+e^{1+\frac{2}{n}}+e^{1+\frac{3}{n}}+\cdots+e^{1+\frac{n}{n}}\right)$?

(A) $\int_0^1 e^x\,dx$

(B) $\int_1^2 e^x\,dx$

(C) $\int_1^2 e^{1+x}\,dx$

(D) $\int_0^2 e^{1+x}\,dx$

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8. The expression $\frac{3}{7}\left(\frac{3}{7}\sin\left(\frac{3}{7}\right)+\frac{6}{7}\sin\left(\frac{6}{7}\right)+\frac{9}{7}\sin\left(\frac{9}{7}\right)+\cdots+\frac{21}{7}\sin\left(\frac{21}{7}\right)\right)$ is a Riemann sum approximation of which of the following integrals?

(A) $\int_0^3 (x\sin x)\,dx$

(B) $\frac{1}{7}\int_0^3 (x\sin x)\,dx$

(C) $\frac{1}{7}\int_0^{21} (\sin x)\,dx$

(D) $\int_0^3 (\sin x)\,dx$

9. The expression $\frac{1}{5}\left(\ln\left(2+\frac{1}{5}\right)+\ln\left(2+\frac{2}{5}\right)+\ln\left(2+\frac{3}{5}\right)+\ln\left(2+\frac{4}{5}\right)+\ln\left(2+\frac{5}{5}\right)\right)$ is a Riemann sum approximation of which of the following integrals?

(A) $\int_2^3 \ln\left(\frac{x}{5}\right) dx$

(B) $\frac{1}{5}\int_0^5 \ln x\, dx$

(C) $\frac{1}{5}\int_2^3 \ln x\, dx$

(D) $\int_2^3 \ln x\, dx$

(E) $\int_0^5 \ln(2+x)\, dx$

6.3 Summation Notation

Test Prep

10. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+\frac{4k}{n}\right)^2\frac{4}{n}$

I. $\int_{-1}^{3} x^2\, dx$

II. $\int_0^4 (-1+x)^2\, dx$

III. $\int_0^1 4(-1+4x)^2\, dx$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III only