6.3 Summation Notation

Calculus

Solutions

Practice

Write a definite integral that is equivalent to the given summation notation. The lower limit for the integral is also given to help you get started.

1. Integral's lower limit = 0

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{n}\right)\left(\cos\left(\frac{\pi}{n}k\right)\right)$$

$$\int_0^{\pi}\cos(x)\,dx$$

2. Integral's lower limit = -3

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{5}{n}\right)\left(\sqrt[3]{-3+\frac{5k}{n}}\right)$$

$$\int_{-3}^{2}\sqrt[3]{x}\,dx$$

3. Integral's lower limit = 6

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{9}{n}\right)\left(\frac{1}{\left(6+\frac{9k}{n}\right)^2}\right)$$

$$\int_6^{15}\frac{1}{x^2}\,dx$$

Write a summation notation equivalent to the definite integral.

4. $\int_{-3}^{3}x^2\,dx$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{6}{n}\right)\left(-3+\frac{6k}{n}\right)^2$$

5. $\int_2^5\frac{1}{x}dx$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{3}{n}\right)\left(\frac{1}{2+\frac{3k}{n}}\right)$$

6. $\int_0^7\sqrt{x}\,dx$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{7}{n}\right)\left(\sqrt{\frac{7k}{n}}\right)$$

7. Which of the following expressions is equal to $\lim_{n\to\infty}\frac{1}{n}\left(e^{1+\frac{1}{n}}+e^{1+\frac{2}{n}}+e^{1+\frac{3}{n}}+\cdots+e^{1+\frac{n}{n}}\right)$?

B

(A) $\int_0^1 e^x\,dx$

(B) $\int_1^2 e^x\,dx$

(C) $\int_1^2 e^{1+x}\,dx$

(D) $\int_0^2 e^{1+x}\,dx$

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8. The expression $\frac{3}{7}\left(\frac{3}{7}\sin\left(\frac{3}{7}\right)+\frac{6}{7}\sin\left(\frac{6}{7}\right)+\frac{9}{7}\sin\left(\frac{9}{7}\right)+\cdots+\frac{21}{7}\sin\left(\frac{21}{7}\right)\right)$ is a Riemann sum approximation of which of the following integrals?

(A) $\int_0^3(x\sin x)\,dx$

(B) $\frac{1}{7}\int_0^3(x\sin x)\,dx$

(C) $\frac{1}{7}\int_0^{21}(\sin x)\,dx$

(D) $\int_0^3(\sin x)\,dx$

9. The expression $\frac{1}{5}\left(\ln\left(2+\frac{1}{5}\right)+\ln\left(2+\frac{2}{5}\right)+\ln\left(2+\frac{3}{5}\right)+\ln\left(2+\frac{4}{5}\right)+\ln\left(2+\frac{5}{5}\right)\right)$ is a Riemann sum approximation of which of the following integrals?

(A) $\int_2^3 \ln\left(\frac{x}{5}\right) dx$

(B) $\frac{1}{5}\int_0^5 \ln x \, dx$

(C) $\frac{1}{5}\int_2^3 \ln x \, dx$

(D) $\int_2^3 \ln x \, dx$

(E) $\int_0^5 \ln(2+x) \, dx$

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Test Prep

10. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+\frac{4k}{n}\right)^2\frac{4}{n}$

I. $\int_{-1}^3 x^2 \, dx$

II. $\int_0^4 (-1+x)^2 \, dx$

III. $\int_0^1 4(-1+4x)^2 \, dx$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III only