Calculus

6.4 Accumulation Functions

Notes

Write your questions and thoughts here!

Last lesson, we learned about the definite integral. $\int_a^b f(x)\,dx$ represents the area under the curve of $f(x)$ on the interval $[a, b]$.

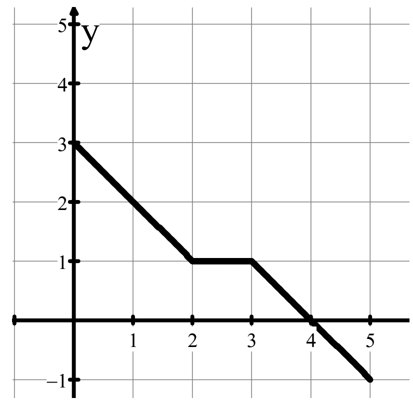
Let us say we know the interval starts at a, but we do not know where it stops. That would give us where a is a constant and x is some unknown variable. We can represent that as a new function that looks like this:

$$F(x) =$$

1. Let $F(x) = \int_0^x f(t)\,dt$. Use the graph of f in the figure to find the values of the table on the interval $0 \le x \le 5$.

   a) Complete the table.

x	0	1	2	3	4	5
$F(x)$						



This is called an ___________________________

Fundamental Theorem of Calculus

If a is a constant and f is a continuous function, then

$$\frac{d}{dx}\int_a^x f(t)\,dt =$$

Derivatives and Integrals are __________ of each other. They cancel each other out, just like multiplication and division. In example 1, the graph of f is the derivative of $F(x)$. So $F(x)$ is considered the _________________ of $f(x)$.

Variations of the FTC

If a is a constant, f is a continuous function, and g and h are differentiable then

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt =$$

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt =$$

Write your questions and thoughts here!

Find $F'(x)$.

1. $F(x) = \int_2^x (3t^2 + 4t)\,dt$	2. $F(x) = \int_{\frac{\pi}{2}}^{x^3} \sin(t)\,dt$	3. $F(x) = \int_1^{4x} h(t)\,dt$
4. $F(x) = \int_{-x}^{x} 5t\,dt$	5. $F(x) = \int_{2x}^{3x} (t^2 - t)\,dt$	

6.4 Accumulation Functions

Calculus

Practice

Find $F'(x)$.

1. $F(x) = \int_2^x t^3\,dt$	2. $F(x) = \int_0^x 5\,dt$	3. $F(x) = \int_{-1}^{x} (4t - t^2)\,dt$
4. $F(x) = \int_\pi^x \cos(t)\,dt$	5. $F(x) = \int_1^{x^2} t^3\,dt$	6. $F(x) = \int_\pi^{x^2} \sin(t)\,dt$
7. $F(x) = \int_\pi^{\sin x} \frac{1}{t}\,dt$	8. $F(x) = \int_4^{x^2} 3\sqrt{t}\,dt$	9. $F(x) = \int_0^{3x} 2t\,dt$
10. $F(x) = \int_0^{\tan x} t^2\,dt$	11. $F(x) = \int_3^{x^2} \tan(t)\,dt$	12. $F(x) = \int_3^{g(x)} \sec(t)\,dt$

13. $F(x) = \int_{1}^{2x} f(t)\,dt$

14. $F(x) = \int_{x}^{x+2} (4t+1)\,dt$

15. $F(x) = \int_{-x^2}^{x} (3t-1)\,dt$

16. $F(x) = \int_{-x}^{x} t^3\,dt$

17. $F(x) = \int_{2x}^{3x} t^2\,dt$

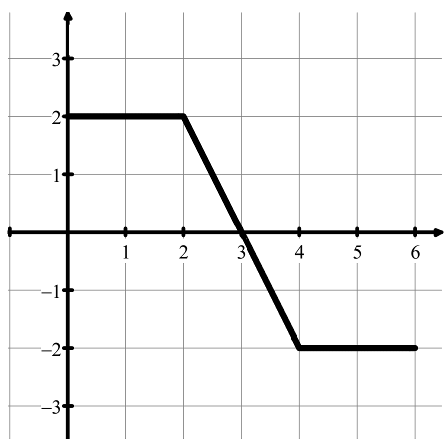
6.4 Accumulation Functions

Test Prep

18. Let $g(x) = \frac{d}{dx}\int_{0}^{x} \sqrt{t^2+9}\,dt$. What is $g(-4)$?

(A) -5 (B) -3 (C) 3 (D) 4 (E) 5

19.

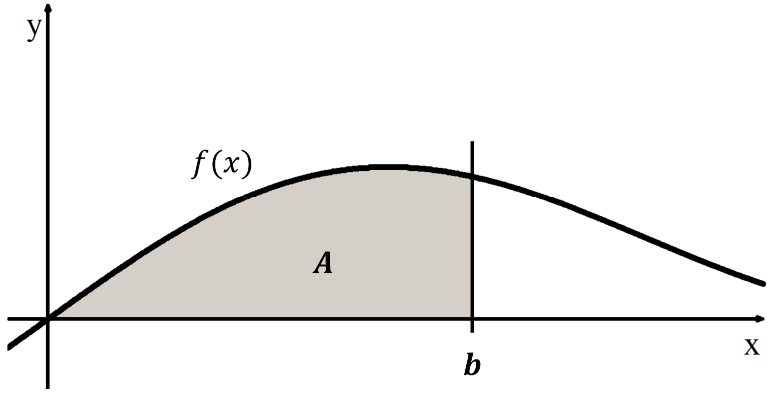


The graph of a function f on the closed interval $[0, 6]$ is shown above. Let $h(x) = \int_{0}^{x} f(t)\,dt$ for $0 \le x \le 6$. Find $h'(3)$.

(A) -2 (B) 0

(C) 2 (D) Does not exist

20.



The figure above shows the region A, which is bounded by the x- and y-axes, the graph of $f(x) = \frac{1-\cos x}{x}$ for $x > 0$, and the vertical line $x = b$. If b increases at a rate of $\frac{\pi}{2}$ units per second, how fast is the area of region A increasing when $b = \frac{\pi}{3}$?