6.4 Accumulation Functions

Calculus

Solutions

Practice

Find $F'(x)$.		
1. $F(x) = \int_2^x t^3\,dt$ $\boxed{x^3}$	2. $F(x) = \int_0^x 5\,dt$ $\boxed{5}$	3. $F(x) = \int_{-1}^x (4t - t^2)\,dt$ $\boxed{4x - x^2}$
4. $F(x) = \int_\pi^x \cos(t)\,dt$ $\boxed{\cos(x)}$	5. $F(x) = \int_1^{x^2} t^3\,dt$ $(x^2)^3 \cdot 2x$ $2x^7$	6. $F(x) = \int_\pi^{x^2} \sin(t)\,dt$ $\sin(x^2) \cdot 2x$ $\boxed{2x \sin(x^2)}$
7. $F(x) = \int_\pi^{\sin x} \frac{1}{t}\,dt$ $\frac{1}{\sin x} \cdot \cos x = \boxed{\cot x}$	8. $F(x) = \int_4^{x^2} 3\sqrt{t}\,dt$ $3\sqrt{x^2} \cdot 2x = \boxed{6x^2}$	9. $F(x) = \int_0^{3x} 2t\,dt$ $2(3x) \cdot 3$ $\boxed{18x}$
10. $F(x) = \int_0^{\tan x} t^2\,dt$ $(\tan x)^2 \cdot \sec^2 x$ $\boxed{\tan^2 x \sec^2 x}$	11. $F(x) = \int_3^{x^2} \tan(t)\,dt$ $\tan(x^2) \cdot 2x$ $\boxed{2x \tan(x^2)}$	12. $F(x) = \int_3^{g(x)} \sec(t)\,dt$ $\boxed{\sec[g(x)] \cdot g'(x)}$

13. $F(x) = \int_1^{2x} f(t)\,dt$

$f(2x)\cdot 2$

$\boxed{2f(2x)}$

14. $F(x) = \int_x^{x+2} (4t+1)\,dt$

$4(x+2)+1-(4x+1)$

$4x+9-4x-1$

$\boxed{8}$

15. $F(x) = \int_{-x^2}^{x} (3t-1)\,dt$

$3x-1-(3(-x^2)-1)\cdot(-2x)$

$3x-1-(6x^3+2x)$

$\boxed{-6x^3+x-1}$

16. $F(x) = \int_{-x}^{x} t^3\,dt$

$x^3-(-x)^3\cdot(-1)$

$x^3-x^3 = \boxed{0}$

17. $F(x) = \int_{2x}^{3x} t^2\,dt$

$(3x)^2\cdot 3-(2x)^2\cdot 2$

$27x^2-8x^2 = \boxed{19x^2}$

6.4 Accumulation Functions

Test Prep

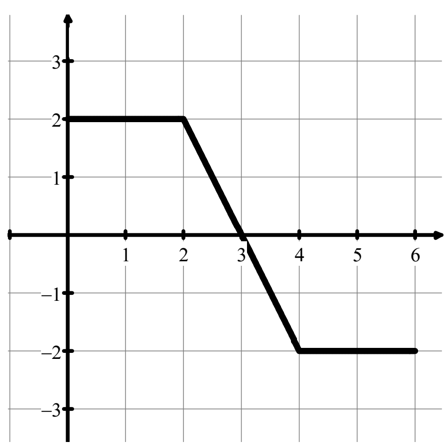
18. Let $g(x) = \frac{d}{dx}\int_0^x \sqrt{t^2+9}\,dt$. What is $g(-4)$?

$g(x)=\sqrt{x^2+9}$

$g(-4)=\sqrt{(-4)^2+9}$

(A) -5 (B) -3 (C) 3 (D) 4 (E) 5

19.



The graph of a function f on the closed interval $[0, 6]$ is shown above. Let $h(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 6$. Find $h'(3)$.

$h'(x)=f(x)$

$h'(3)=f(3)$

B

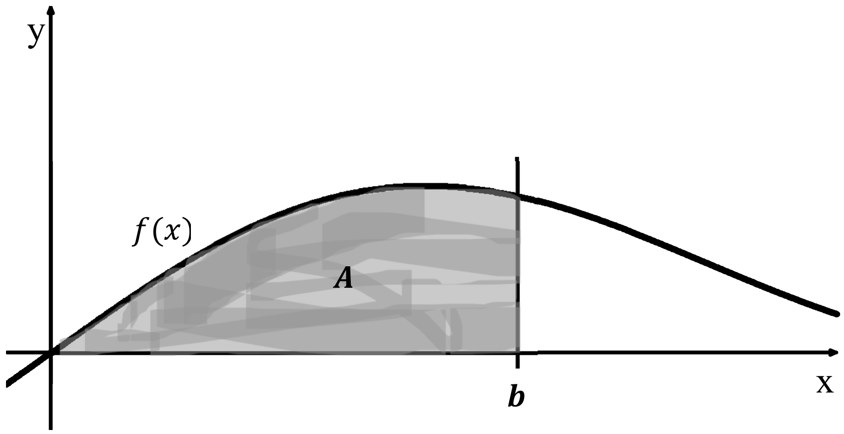
(A) -2

(B) 0

(C) 2

(D) Does not exist

20.



The figure above shows the region A, which is bounded by the x- and y-axes, the graph of $f(x) = \frac{1-\cos x}{x}$ for $x > 0$, and the vertical line $x = b$. If b increases at a rate of $\frac{\pi}{2}$ units per second, how fast is the area of region A increasing when $b = \frac{\pi}{3}$?

$\frac{db}{dt} = \pi/2$ $\quad b = \pi/3 \quad$ $\frac{dA}{dt} = ?$

$$A = \int_0^b \frac{1-\cos x}{x}\, dx$$

$$\frac{dA}{dt} = \frac{1-\cos b}{b} \frac{db}{dt}$$

$$\frac{dA}{dt} = \frac{1-\cos \pi/3}{\pi/3} \cdot \left(\frac{\pi}{2}\right)$$

$$\frac{dA}{dt} = \frac{1-\frac{1}{2}}{\pi/3} \cdot \pi/2$$

$$\frac{dA}{dt} = \frac{1}{2} \cdot \frac{3}{\pi} \cdot \frac{\pi}{2} = \boxed{\frac{3}{4}}$$