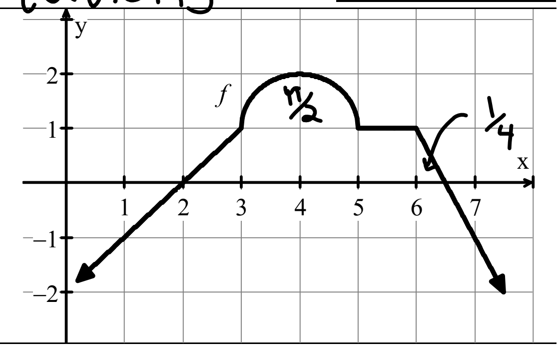
6.5 Behavior of Accumulation Functions

Calculus

Solutions

Practice

1. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.



a. Relative minimum(s)

$x = 2$

b. Relative maximum(s)

$x = 6.5$

c. Concave up

$(-\infty, 4)$

d. Concave down

$(4, 5) \cup (6, \infty)$

e. Increasing

$(2, 6.5)$

f. Decreasing

$(-\infty, 2) \cup (6.5, \infty)$

g. Point(s) of inflection

$x = 4$

h. If $g(3) = -2$, what is the maximum value of g on the interval $[3, 7]$?

$g(6.5) = -2 + \int_3^{6.5} f(t)$

$-2 + \left(\frac{\pi}{2} + 2 + 1 + \frac{1}{4}\right)$

$\boxed{\frac{5}{4} + \frac{\pi}{2}}$

i. Given $h(x) = \int_0^{2x-7} f(t)\,dt$. Find the x-value where h has a relative minimum.

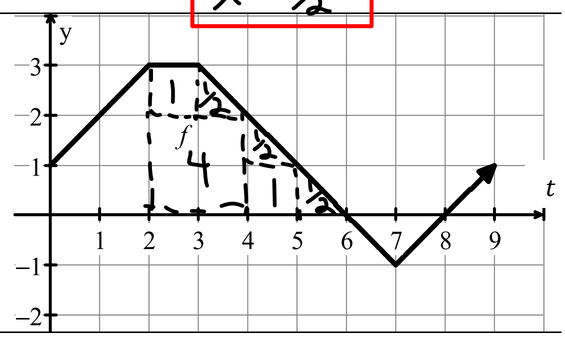
$h'(x) = f(2x-7) \cdot 2 = 0$

$f(2x-7) = 0$

$2x - 7 = 2$

$\boxed{x = \frac{9}{2}}$

2. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.



a. Relative minimum(s)

$x = 8$

b. Relative maximum(s)

$x = 6$

c. Concave up

$(0, 2) \cup (7, \infty)$

d. Concave down

$(3, 7)$

e. Increasing

$(0, 6) \cup (8, \infty)$

f. Decreasing

$(6, 8)$

g. Point(s) of inflection

$x = 7$

h. If $g(2) = 1$, what is the maximum value of g on the interval $[2, 9]$?

$g(6) = 1 + \int_2^6 f(t)\,dt$

$1 + 4 + 2 + 1\frac{1}{2}$

$\boxed{8.5}$

i. Given $h(x) = \int_0^{\frac{x}{2}+5} f(t)\,dt$. Find the x-value where h has a relative minimum.

$h'(x) = f\left(\frac{x}{2} + 5\right) \cdot \frac{1}{2} = 0$

$\frac{x}{2} + 5 = 8$

$\frac{x}{2} = 3$

$\boxed{x = 6}$

3. **Calculator active problem.** Let f be the function given by $f(x) = \int_{1/10}^{x} \sin\left(\frac{1}{t}\right) dt$ for $\frac{1}{10} < x < 1$. At what value(s) of x does f attain a relative maximum?

$x \simeq 0.159$

4. **Calculator active problem.** Let h be the function given by $h(x) = \int_{1}^{x} (1 - e^{\cos t})\, dt$ for $1 < x < 10$. At what value(s) of x does h attain a relative minimum?

$x \simeq 1.5707$ and 7.8539

5.

x	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$	4	$4 < x < 5$
$f(x)$	2	Positive	0	Negative	-3	Negative	0	Positive
$f'(x)$	-1	Negative	0	Negative	DNE	Positive	0	Negative
$f''(x)$	1	Positive	0	Negative	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[1, 5)$. The function f is twice differentiable except at $x = 3$. The function f and its derivatives have the properties indicated in the table above.

Let g be the function defined by $g(x) = \int_{2}^{x} f(t)\, dt$ on the open interval $(1, 5)$.

a. For $1 < x < 5$, find all critical points of g.

C.P. of g when g' (or $f(x)$) $= 0$ or DNE

$\boxed{x=2 \quad x=4}$

b. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

Rel. max at $x=2$ because g' changes sign from pos. to neg.

Rel. min at $x=4$ because g' changes sign from neg to pos.

c. For $1 < x < 5$, find all values of x at which g has a point of inflection.

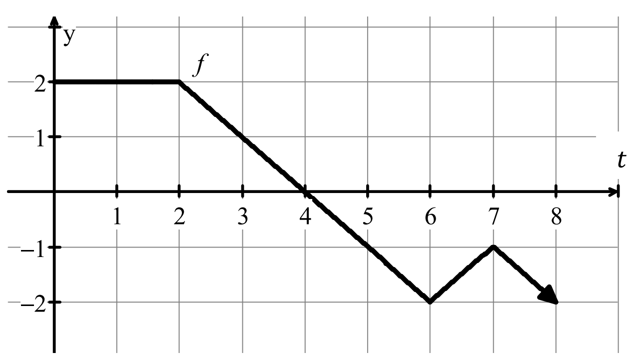
P.o.i. when g'' (or $f'(x)$) changes sign

$\boxed{x=3 \text{ and } x=4}$

Test Prep

6.5 Behavior of Accumulation Functions

6.



The graph of the function f is shown above. Let $g(x) = \int_{0}^{x} f(t)\, dt$.

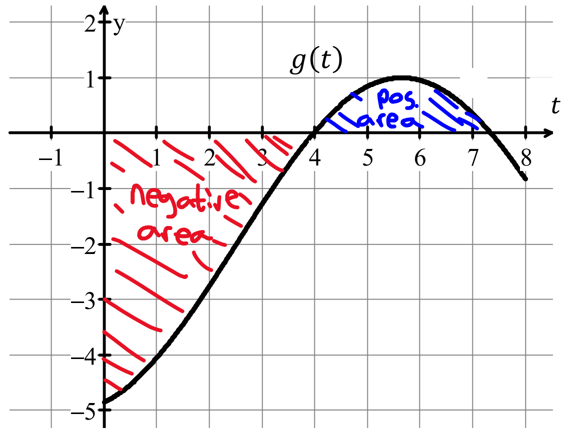
a. Find the value of $g'(6)$.

$g'(6) = f(6) = \boxed{-2}$

b. Find the value of $g''(6)$.

$g''(6) = f'(6) =$ corner $\rightarrow$ $\boxed{\text{Does not exist.}}$

7.



The graph of a differentiable function g is shown above. If $h(x) = \int_0^x g(t)\,dt$, which of the following is true?

(A) $h(4) < h'(4) < h''(4)$

(B) $h(4) < h''(4) < h'(4)$

(C) $h'(4) < h(4) < h''(4)$

(D) $h''(4) < h(4) < h'(4)$

(E) $h''(4) < h'(4) < h(4)$

$h(4) = \int_0^4 g(t) =$ area < 0

$h'(4) = g(4) = 0$

$h''(4) = g'(4) =$ slope > 0