6.6 Properties of Definite Integrals

1. $\int_0^{12} f(x)\, dx =$

2. $\int_{12}^0 f(x)\, dx =$

Properties of Definite Integrals

Equivalent Limits	Reversal of Limits
$\displaystyle\int_a^a f(x)\, dx =$	$\displaystyle\int_b^a f(x)\, dx =$
Multiply by constant $(k = \text{constant})$	**Adjacent Intervals** $(a < c < b)$
$\displaystyle\int_a^b k f(x)\, dx =$	$\displaystyle\int_a^c f(x)\, dx + \int_c^b f(x)\, dx =$
Addition	**Subtraction**
$\displaystyle\int_a^b [f(x) + g(x)]\, dx =$	$\displaystyle\int_a^b [f(x) - g(x)]\, dx =$
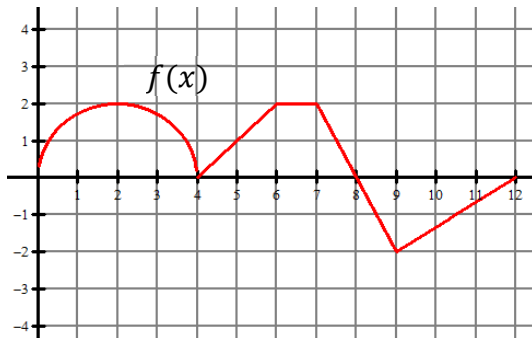
3. $\int_7^6 f(x)\, dx =$

4. $\int_{12}^8 3f(x)\, dx =$

5. Given that $\int_{-2}^1 f(x)\, dx = 4$, $\int_1^5 f(x)\, dx = -3$, and $\int_{-2}^1 g(x)\, dx = 8$, find the following.

a. $\int_5^1 f(x)\, dx$

b. $\int_{-2}^5 f(x)\, dx$

c. $\int_{-2}^1 [f(x) + 2g(x)]\, dx$

d. $\int_0^1 f(x)\, dx$
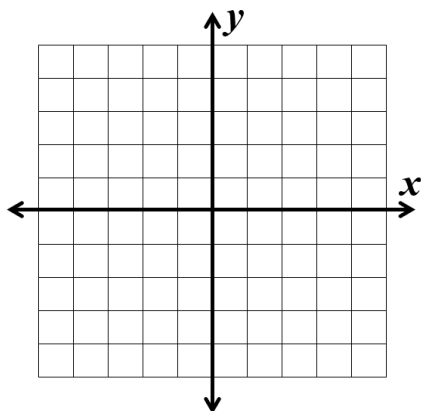
e. $\int_1^{-2} 3f(x)\, dx$

f. $\int_5^5 [f(x) - g(x)]\, dx$

Piecewise-functions and integrals

6. The function g is defined by
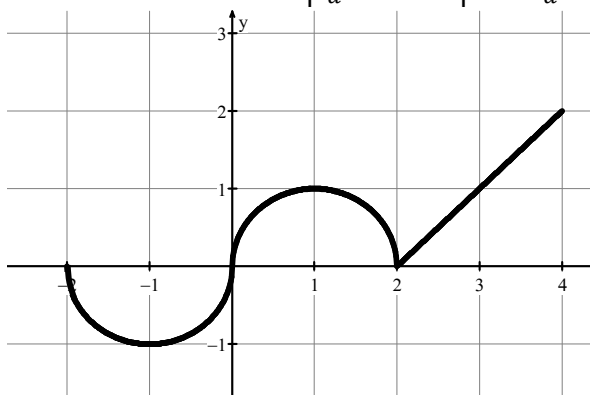$$g(x) = \begin{cases} 3 & \text{for } x < 2 \\ 4 - x & \text{for } x \geq 2 \end{cases}$$
What is the value of $\int_1^5 g(x)\, dx$?

7. What is the value of $\int_0^5 |x - 2|\, dx$?
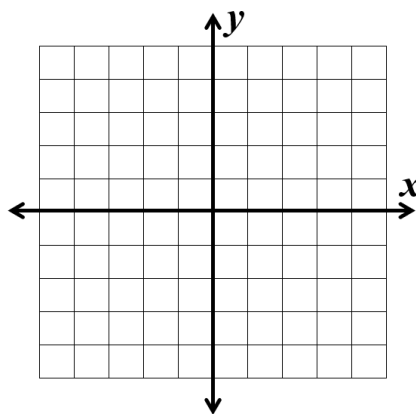
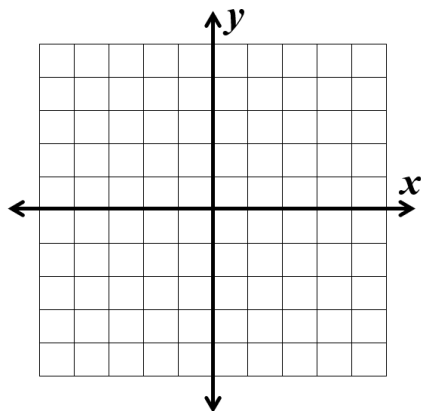What is the difference between $\left| \int_a^b f(x)\, dx \right|$ and $\int_a^b |f(x)|\, dx$?

8. $\left| \int_{-2}^4 f(x)\, dx \right| =$
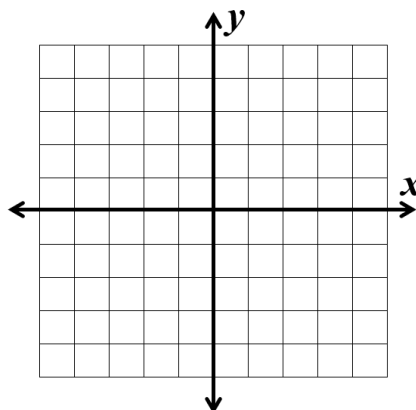
9. $\int_{-2}^4 |f(x)|\, dx =$

Using a calculator to find an integral value

Sketch a graph of the definite integral. Use the calculator to evaluate.

10. $\int_2^3 \sqrt{x - 1}\, dx$

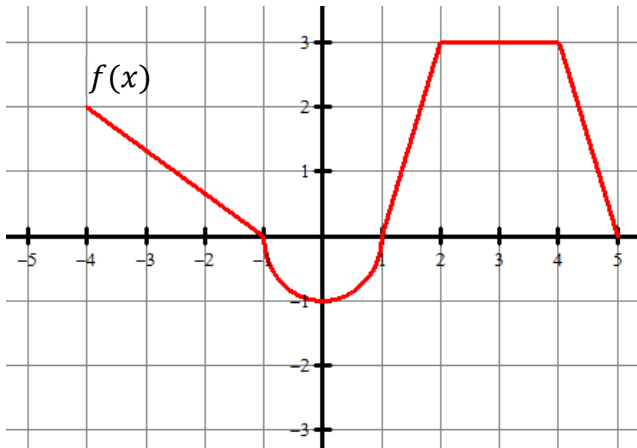11. $\int_{-2}^4 \left(\frac{x}{3} - 1 \right) dx$

6.6 Properties of Definite Integrals

Calculus

The graph of f consists of line segments and a semicircle. Evaluate each definite integral.

1.

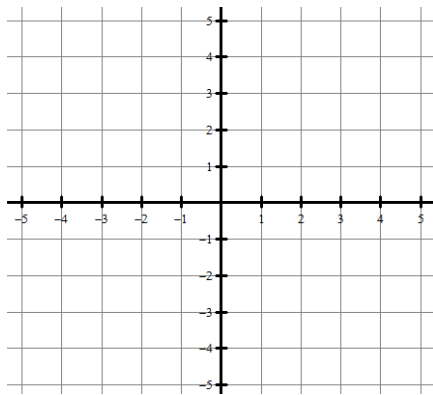a. $\int_{-4}^{-1} f(x)\,dx =$

b. $\int_{2}^{1} f(x)\,dx =$

c. $\int_{1}^{5} 2f(x)\,dx =$

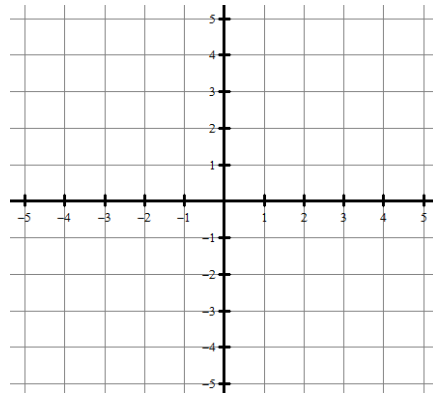d. $\int_{-4}^{5} f(x)\,dx =$

e. $\int_{4}^{2} f(x)\,dx =$

f. $\int_{-4}^{1} |f(x)|\,dx =$

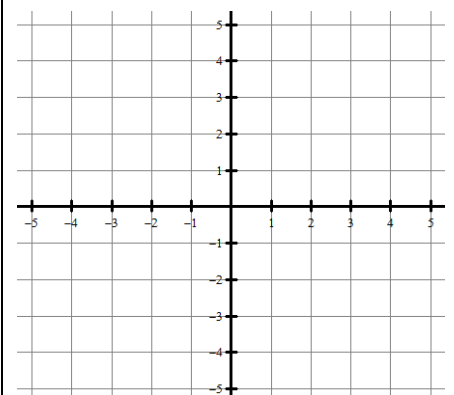**Sketch a graph of the definite integral. Evaluate the integral with a graphing calculator.**

2. $\int_{0}^{3} -\sqrt{x+1}\,dx =$

3. $\int_{-2}^{3} |x+1|\,dx =$

4. $\int_{1}^{-3} \left(-\frac{x}{2}+1\right) dx =$

Let f and g be continuous functions that produce the following definite integral values.

$$\int_{-3}^{2} f(x)\,dx = 2 \qquad \int_{2}^{7} f(x)\,dx = -5 \qquad \int_{-3}^{2} g(x)\,dx = 6$$

Find the following.

5. $\int_{2}^{7} 2f(x)\,dx$

6. $4\int_{-3}^{2} f(x)\,dx$

7. $\int_{-3}^{7} f(x)\,dx$

8. $\int_{2}^{-3} g(x)\,dx$

9. $\int_{-3}^{2} [g(x) - f(x)]\,dx$

10. $\left| \int_{2}^{7} f(x)\,dx \right|$

11. $-\int_{7}^{2} f(x)\,dx$

$$\int_1^2 f(x)\,dx = -2 \qquad \int_1^6 f(x)\,dx = 4 \qquad \int_1^6 g(x)\,dx = 8$$

Find the following.

12. $\int_2^2 g(x)\,dx$	13. $\int_6^1 g(x)\,dx$	14. $3\int_1^2 f(x)\,dx$	15. $\int_2^6 f(x)\,dx$
16. $\int_1^6 [f(x) - g(x)]\,dx$	17. $\int_1^6 [3f(x) - g(x)]\,dx$	18. $\int_1^6 |f(x) - g(x)|\,dx$	19. $\left|\int_1^6 f(x) - g(x)\,dx\right|$

6.6 Properties of Definite Integrals

Test Prep

20.
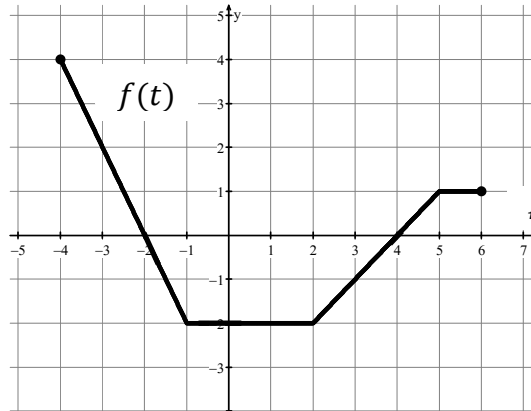
The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_2^x f(t)\,dt$.

a. Find the average rate of change of g from $x = -4$ to $x = 6$.

b. Find the instantaneous rate of change of g with respect to x at $x = 5$, or state that it does not exist.

c. On what open intervals, if any, is the graph of g concave down? Justify your answer.

d. Find all x-values in the interval $-4 < x < 6$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.