## 6.6 Properties of Definite Integrals

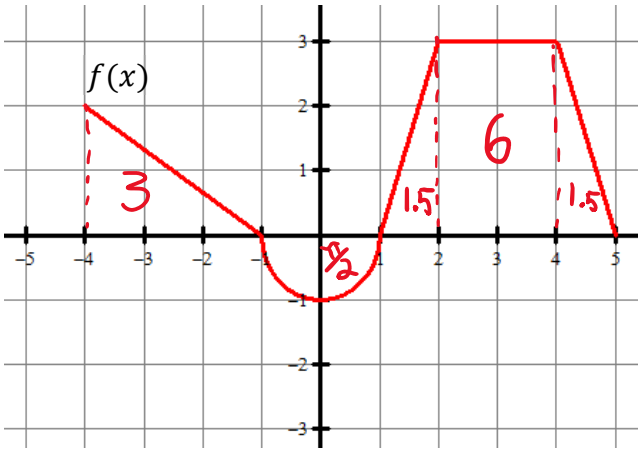
Calculus

Solutions

**Practice**

**The graph of $f$ consists of line segments and a semicircle. Evaluate each definite integral.**

1.




a. $\int_{-4}^{-1} f(x)\,dx =$ 3

b. $\int_{2}^{1} f(x)\,dx =$ -1.5

c. $\int_{1}^{5} 2f(x)\,dx =$ 18

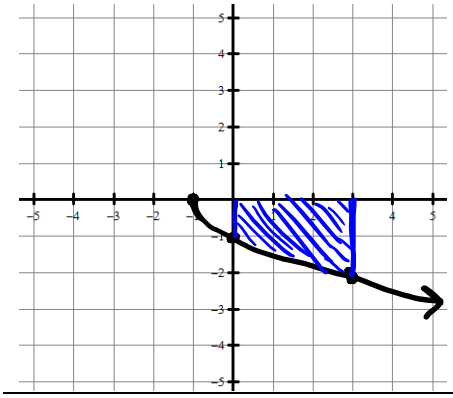
d. $\int_{-4}^{5} f(x)\,dx =$ $12-\pi/2$

e. $\int_{4}^{2} f(x)\,dx =$ -6

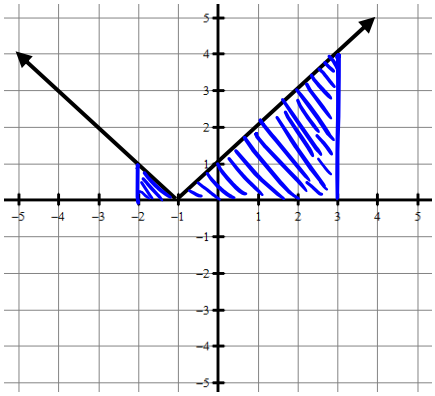
f. $\int_{-4}^{1} |f(x)|\,dx =$ $3+\pi/2$

**Sketch a graph of the definite integral. Evaluate the integral with a graphing calculator.**

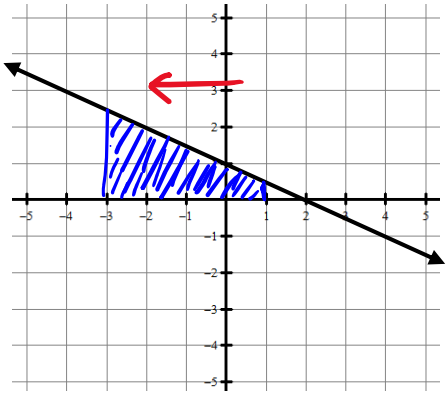
2. $\int_{0}^{3} -\sqrt{x+1}\,dx =$ -4.6667



3. $\int_{-2}^{3} |x+1|\,dx =$ 8.5



4. $\int_{1}^{-3} \left(-\frac{x}{2}+1\right) dx =$ -6



**Let $f$ and $g$ be continuous functions that produce the following definite integral values.**

$$\int_{-3}^{2} f(x)\,dx = 2 \qquad \int_{2}^{7} f(x)\,dx = -5 \qquad \int_{-3}^{2} g(x)\,dx = 6$$

Find the following.

5. $\int_{2}^{7} 2f(x)\,dx$ → -10

6. $4\int_{-3}^{2} f(x)\,dx$ → 8

7. $\int_{-3}^{7} f(x)\,dx$ → $2 + -5$ → -3

8. $\int_{2}^{-3} g(x)\,dx$ → -6

9. $\int_{-3}^{2} [g(x) - f(x)]\,dx$ → $6 - 2$ → 4

10. $\left|\int_{2}^{7} f(x)\,dx\right|$ → 5

11. $-\int_{7}^{2} f(x)\,dx$ → $\int_{2}^{7} f(x)\,dx$ → -5

**Let $f$ and $g$ be continuous functions that produce the following definite integral values.**

$$\int_1^2 f(x)\,dx = -2 \qquad \int_1^6 f(x)\,dx = 4 \qquad \int_1^6 g(x)\,dx = 8$$

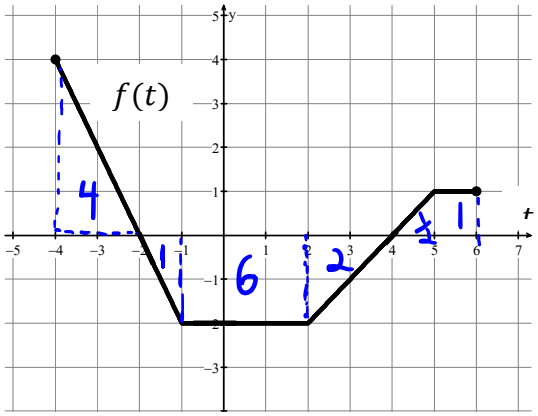
Find the following.

| | | | |
|---|---|---|---|
| 12. $\int_2^2 g(x)\,dx$ <br> 0 | 13. $\int_6^1 g(x)\,dx$ <br> $-8$ | 14. $3\int_1^2 f(x)\,dx$ <br> $-6$ | 15. $\int_2^6 f(x)\,dx$ <br> $\int_1^6 f(x) - \int_1^2 f(x)$ <br> $4 - -2$ <br> 6 |
| 16. $\int_1^6 [f(x) - g(x)]\,dx$ <br> $4 - 8$ <br> $-4$ | 17. $\int_1^6 [3f(x) - g(x)]\,dx$ <br> $12 - 8$ <br> 4 | 18. $\int_1^6 \lvert f(x) - g(x)\rvert\,dx$ <br> cannot be determined | 19. $\left\lvert \int_1^6 f(x) - g(x)\,dx \right\rvert$ <br> 4 |

## 6.6 Properties of Definite Integrals

**Test Prep**

20.




The graph of the function $f$ is shown above. Let $g$ be the function defined by $g(x) = \int_2^x f(t)\,dt$.

a. Find the average rate of change of $g$ from $x = -4$ to $x = 6$.

$$\frac{g(6) - g(-4)}{6 - -4} = \frac{(-2 + 1.5) - (7 - 4)}{10} = \frac{-\frac{1}{2} - 3}{10} = -\frac{7}{2} \cdot \frac{1}{10} = -\frac{7}{20}$$

b. Find the instantaneous rate of change of $g$ with respect to $x$ at $x = 5$, or state that it does not exist.

$g'(5) = f(5) = 1$

c. On what open intervals, if any, is the graph of $g$ concave down? Justify your answer.

Concave down on $(-4, -1)$ because $g''(x)$ [which equals $f'(x)$] is negative

d. Find all $x$-values in the interval $-4 < x < 6$ at which $g$ has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

$x = -2$ is a relative max b/c $g'$ changes sign from pos to neg.

$x = 4$ is a relative min b/c $g'$ changes sign from neg. to pos.