Calculus

Write your questions and thoughts here!

6.7 Definite Integrals

Notes

An **antiderivative** of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$.

For example, let $f(x) = 3x^2$ and $f'(x) = 6x$. The expression $3x^2$ is an antiderivative of $6x$. Why?

When we take an integral, it is taking the **anti**derivative of a function. The area under the curve is represented by an antiderivative! What?!

Power Rule for finding a derivative.	**Antiderivative is the reverse order.**
$f(x) = x^n$ $f'(x) =$ **Step one:** Multiply by the old exponent. **Step two:** Subtract one from the exponent.	$f(x) = x^n$ $F(x) =$ **Step one:** **Step two:**

Given the function $f(x)$, find the antiderivative $F(x)$.

1. $f(x) = x^2$	2. $f(x) = 5x^3 + \frac{6}{x^3} - 1$	3. $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$

Sine and Cosine Integrals

$$\int \sin x \, dx =$$

$$\int \cos x \, dx =$$

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The Fundamental Theorem of Calculus

If f is continuous on the interval $[a, b]$, then the area under the curve of f from $[a, b]$ can be represented by

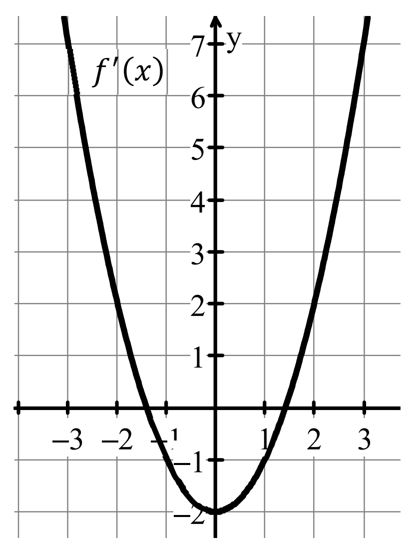
$$\int_a^b f(x)\,dx =$$

where $F(x)$ is the antiderivative of f.

Evaluate the definite integral. Use a calculator to check your answer.

4. $\int_{-2}^{5}(4-6x)\,dx$	5. $\int_1^4\left(\sqrt{x}-\frac{1}{x^2}\right)dx$	6. $\int_0^{\frac{\pi}{2}} 4\sin(x)\,dx$

7. If $f'(x) = x^2 - 2$ and $f(1) = -2$, then $f(3) =$



6.7 Definite Integrals

Calculus

Practice

Find the value of the definite integral. Use a calculator to check your answer.

1. $\int_0^4 (2x+4)\,dx$

2. $\int_0^{\frac{\pi}{2}} (\sin x - x)\,dx$

3. $\int_{-1}^{3} (6x^2 - 8)\,dx$

4. $\int_4^9 \frac{1}{\sqrt{x}}\,dx$

5. $\int_{-4}^{-1} \left(\frac{3}{x^2} + 1\right) dx$

6. $\int_{-\frac{\pi}{2}}^{0} (2 - \cos x)\,dx$

For # 7-13, use the given information to find the value of the function.

7. If $f'(x) = \cos x$ and $f(-\pi) = 12$, then $f\left(\frac{3\pi}{2}\right) =$

8. **Calculator active.** If $f'(x) = \sin(3x) + e^x$ and $f(1) = 0.751$, then $f(4) =$

9. Let f be a differentiable function such that $f(1) = 4$ and $f'(x) = 6x^2 + 3$. What is the value of $f(3)$?

10. **Calculator active.** Let f be a differentiable function such that $f(0) = -0.5$ and $f'(x) = 2 - \cos(ex)$. What is the value of $f(-2)$?

11. Let $h(x)$ be an antiderivative of $5 - 3x$. If $h(-1) = -3$, then $h(2) =$

12. **Calculator active.** Let $F(x)$ be an antiderivative of $\frac{\ln x}{x}$. If $F(2) = -0.13$, then $F(5) =$

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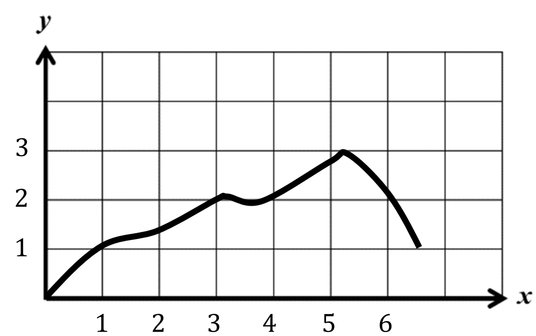
Test Prep

13. The graph of f is shown in the figure to the right. If $\int_0^3 f(x)\,dx = 3.5$ and $F'(x) = f(x)$, then $F(4) - F(0) =$



(A) 6.5 (B) 1.5 (C) 2.5 (D) 5.5 (E) 4.5

14. **Calculator active problem.** Let $f(x) = \int_0^{x^2} \cos t \, dt$. At how many points in the closed interval $[-\sqrt{\pi}, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero (B) One (C) Two (D) Three (E) Four

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15. Given $h(x) = \begin{cases} x-1 & \text{for } x < 0 \\ \sin x & \text{for } x \geq 0 \end{cases}$, find $\int_{-1}^{\pi} h(x) \, dx$

(A) $\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{3}{2}$ (D) $\frac{1}{2}$ (E) $-\frac{7}{2}$

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16. A cubic polynomial function f is defined by $f(x) = \frac{2}{3}x^3 + ax^2 + bx + c$, where $a, b,$ and c are constants. The function f has a local minimum at $x = -2$, and the graph of f has a point of inflection at $x = -5$. If $\int_0^1 f(x) \, dx = \frac{15}{2}$, what is the value of c?