6.7 Definite Integrals

Calculus

Solutions

Practice

Find the value of the definite integral. Use a calculator to check your answer.

1. $\int_0^4 (2x+4)\,dx$

$\frac{2x^2}{2} + 4x \Big|_0^4$

$(16+16) - (0)$

$\boxed{32}$

2. $\int_0^{\frac{\pi}{2}} (\sin x - x)\,dx$

$-\cos x - \frac{x^2}{2} \Big|_0^{\pi/2}$

$\left[-\cos\left(\frac{\pi}{2}\right) - \frac{1}{2}\cdot\frac{\pi^2}{4}\right] - \left[-\cos(0) - 0\right]$

$\left[0 - \frac{\pi^2}{8}\right] - [-1]$

$\boxed{1 - \frac{\pi^2}{8}}$

3. $\int_{-1}^{3} (6x^2 - 8)\,dx$

$\frac{6x^3}{3} - 8x \Big|_{-1}^{3}$

$(2(3)^3 - 24) - (2(-1)^3 + 8)$

$(54 - 24) - (6)$

$\boxed{24}$

4. $\int_4^9 \frac{1}{\sqrt{x}}\,dx$

$\int_4^9 x^{-1/2}\,dx$

$\frac{x^{1/2}}{1/2} \Big|_4^9$

$(2\cdot 3) - (2\cdot 2)$

$\boxed{2}$

5. $\int_{-4}^{-1} \left(\frac{3}{x^2} + 1\right) dx$

$\int_{-4}^{-1} 3x^{-2} + 1\,dx$

$\frac{3x^{-1}}{-1} + x \Big|_{-4}^{-1}$

$\left[\frac{-3}{(-1)} + (-1)\right] - \left[\frac{-3}{-4} + (-4)\right]$

$[\,2\,] - \left[\frac{3}{4} - \frac{16}{4}\right]$

$\frac{8}{4} + \frac{13}{4}$

$\boxed{\frac{21}{4}}$

6. $\int_{-\frac{\pi}{2}}^{0} (2 - \cos x)\,dx$

$2x - \sin x \Big|_{-\pi/2}^{0}$

$(0 - \sin 0) - \left(2\left(-\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right)$

$[0] - [-\pi + 1]$

$\boxed{\pi - 1}$

For # 7-13, use the given information to find the value of the function.

7. If $f'(x) = \cos x$ and $f(-\pi) = 12$, then $f\left(\frac{3\pi}{2}\right) =$

$12 + \int_{-\pi}^{3\pi/2} \cos x\,dx$

$12 + [\sin x] \Big|_{-\pi}^{3\pi/2}$

$12 + [(-1) - (0)]$

$\boxed{11}$

8. **Calculator active.** If $f'(x) = \sin(3x) + e^x$ and $f(1) = 0.751$, then $f(4) =$

$0.751 + \int_1^4 \left[\sin(3x) + e^x\right] dx$

$\boxed{52.0195}$

9. Let f be a differentiable function such that $f(1) = 4$ and $f'(x) = 6x^2 + 3$. What is the value of $f(3)$?

$4 + \int_1^3 (6x^2+3)\,dx$

$4+[2x^3+3x]\Big|_1^3$

$4+[2(27)+9]-[2+3]$

$4+[63]-[5]$

$\boxed{62}$

10. **Calculator active.** Let f be a differentiable function such that $f(0) = -0.5$ and $f'(x) = 2 - \cos(ex)$. What is the value of $f(-2)$?

$-0.5 + \int_0^{-2} (2-\cos(ex))\,dx$

$\boxed{-4.7755}$

11. Let $h(x)$ be an antiderivative of $5 - 3x$. If $h(-1) = -3$, then $h(2) =$

$-3 + \int_{-1}^2 (5-3x)\,dx$

$-3+\left[5x - \frac{3x^2}{2}\right]\Big|_{-1}^2$

$-3+[10-6]-\left[-5-\frac{3}{2}\right]$

$\boxed{7.5}$

12. **Calculator active.** Let $F(x)$ be an antiderivative of $\frac{\ln x}{x}$. If $F(2) = -0.13$, then $F(5) =$

$-0.13 + \int_2^5 \frac{\ln x}{x}\,dx$

$\boxed{0.9249}$

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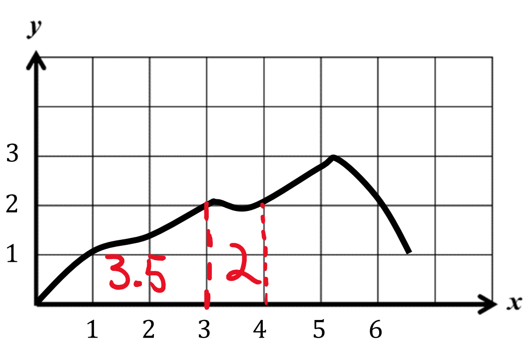
Test Prep

13. The graph of f is shown in the figure to the right. If $\int_0^3 f(x)\,dx = 3.5$ and $F'(x) = f(x)$, then $F(4) - F(0) =$

$\int_0^4 f(x)\,dx$

$\int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx$



(A) 6.5 (B) 1.5 (C) 2.5 (D) 5.5 (E) 4.5

14. **Calculator active problem.** Let $f(x) = \int_0^{x^2} \cos t \, dt$. At how many points in the closed interval $[-\sqrt{\pi}, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

$f'(x) = \cos(x^2) \cdot 2x$

Avg rate $= \dfrac{f(\sqrt{\pi}) - f(-\sqrt{\pi})}{\sqrt{\pi} - -\sqrt{\pi}}$

$\dfrac{\sin \pi - \sin(\pi)}{2\sqrt{\pi}} = 0$

$f(x) = \sin t \Big|_0^{x^2}$

$f(x) = \sin x^2 - \sin 0$

$f(x) = \sin x^2$

$2x\cos(x^2) = 0$

Graph and look for zeros on the interval $[-\sqrt{\pi}, \sqrt{\pi}]$

(A) Zero (B) One (C) Two (D) Three (E) Four

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15. Given $h(x) = \begin{cases} x - 1 & \text{for } x < 0 \\ \sin x & \text{for } x \geq 0 \end{cases}$, find $\int_{-1}^{\pi} h(x)\, dx$

$\int_{-1}^{0} (x-1)\, dx + \int_0^{\pi} \sin x \, dx$

$\left[\frac{x^2}{2} - x\right]\Big|_{-1}^{0} + \left[-\cos x\right]\Big|_0^{\pi}$

$(0) - (\frac{1}{2} + 1) + (-\cos \pi) - (-\cos(0))$

$-\frac{3}{2} + (1) - (-1)$

$\frac{1}{2}$

(A) $\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{3}{2}$ (D) $\frac{1}{2}$ (E) $-\frac{7}{2}$

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16. A cubic polynomial function f is defined by $f(x) = \frac{2}{3}x^3 + ax^2 + bx + c$, where $a, b,$ and c are constants. The function f has a local minimum at $x = -2$, and the graph of f has a point of inflection at $x = -5$. If $\int_0^1 f(x)\, dx = \frac{15}{2}$, what is the value of c?

$f'(x) = 2x^2 + 2ax + b$

$f''(x) = 4x + 2a$

Pt. of inflection $\rightarrow$ $4(-5) + 2a = 0$

$2a = 20$

$a = 10$

Local min $\rightarrow$ $2(-2)^2 + 2(10)(-2) + b = 0$

$8 - 40 + b = 0$

$b = 32$

$\int_0^1 \left(\frac{2}{3}x^3 + 10x^2 + 32x + c\right) dx = \frac{15}{2}$

$\left[\frac{2}{3}\frac{x^4}{4} + \frac{10}{3}x^3 + 16x^2 + cx\right]\Big|_0^1 = \frac{15}{2}$

$\left(\frac{1}{6} + \frac{10}{3} + 16 + c\right) - (0) = \frac{15}{2}$

$\boxed{c = -12}$