6.9 Integrating Using Substitution

Calculus

Solutions

Practice

Find the indefinite integrals.

1. $\int \frac{x^2}{(1+x^3)^2}dx$

$u = 1+x^3$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$\int \frac{x^2}{u^2}\left(\frac{du}{3x^2}\right)$

$\frac{1}{3}\int \frac{1}{u^2}du$

$\frac{1}{3}\frac{u^{-1}}{-1} = -\frac{1}{3u} + C$

$\boxed{-\frac{1}{3+3x^3} + C}$

2. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du = dx$

$\int \frac{\cos u}{\sqrt{x}}(2\sqrt{x}\,du)$

$2\int \cos u\,du$

$2\sin u + C$

$\boxed{2\sin\sqrt{x} + C}$

3. $\int \frac{\sin x}{1+\cos^2 x}dx$

$u = \cos x$

$du = -\sin x\,dx$

$\frac{du}{-\sin x} = dx$

$\int \frac{\sin x}{1+u^2}\left(\frac{du}{-\sin x}\right)$

$-\int \frac{1}{1+u^2}du$

$-\tan^{-1}u + C$

$\boxed{-\tan^{-1}(\cos x) + C}$

4. $\int \frac{1}{\sqrt{1-9x^2}}dx$ ← $\sin^{-1}(x)$

$\int \frac{1}{\sqrt{1-(3x)^2}}dx$

$u = 3x$

$du = 3\,dx$

$\frac{du}{3} = dx$

$\int \frac{1}{\sqrt{1-u^2}}\left(\frac{du}{3}\right)$

$\frac{1}{3}\sin^{-1}(u) + C$

$\boxed{\frac{1}{3}\sin^{-1}(3x) + C}$

5. $\int e^x \sin e^x dx$

$u = e^x$

$du = e^x dx$

$\frac{du}{e^x} = dx$

$\int e^x \sin u\left(\frac{du}{e^x}\right)$

$\int \sin u\,du$

$-\cos u + C$

$\boxed{-\cos(e^x) + C}$

6. $\int \tan x \cos x\,dx$

$\int \frac{\sin x}{\cos x}\cdot \cos x\,dx$

$\int \sin x\,dx$

$\boxed{-\cos x + C}$

7. $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\frac{du}{\sec^2 x} = dx$

$\int \frac{\sec^2 x}{\sqrt{u}} \left(\frac{du}{\sec^2 x}\right)$

$\int u^{-1/2} du$

$2u^{1/2} + C$

$\boxed{2\sqrt{\tan x} + C}$

8. $\int \frac{x\,dx}{\sqrt{1-x^2}}$

$u = 1 - x^2$

$du = -2x \, dx$

$\frac{du}{-2x} = dx$

$\int \frac{x}{\sqrt{u}} \left(\frac{du}{-2x}\right)$

$-\frac{1}{2} \int u^{-1/2} du$

$-\frac{1}{2} \cdot 2u^{1/2} + C$

$\boxed{-\sqrt{1-x^2} + C}$

9. $\int \frac{(\ln x)^5}{x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$x\,du = dx$

$\int \frac{u^5}{x} (x\,du)$

$\int u^5 du$

$\frac{u^6}{6} + C$

$\boxed{\frac{1}{6}(\ln x)^6 + C}$

10. $\int \frac{1}{25x^2+1} dx$ ← $\tan^{-1}(x)$

$\int \frac{1}{(5x)^2+1} dx$

$u = 5x$

$\frac{du}{5} = dx$

$\int \frac{1}{u^2+1} \left(\frac{du}{5}\right)$

$\frac{1}{5} \tan^{-1} u + C$

$\boxed{\frac{1}{5} \tan^{-1}(5x) + C}$

11. $\int (2x+5)(x^2+5x)^7 dx$

$u = x^2 + 5x$

$du = 2x + 5 \, dx$

$\frac{du}{2x+5} = dx$

$\int (2x+5)(u)^7 \left(\frac{du}{2x+5}\right)$

$\int u^7 du$

$\frac{1}{8} u^8 + C$

$\boxed{\frac{1}{8}(x^2+5x)^8 + C}$

12. $\int \frac{e^x}{4-e^x} dx$

$u = 4 - e^x$

$du = -e^x dx$

$\frac{du}{-e^x} = dx$

$\int \frac{e^x}{u} \left(\frac{du}{-e^x}\right)$

$-\int \frac{1}{u} du$

$-\ln|u| + C$

$\boxed{-\ln|4-e^x| + C}$

Evaluate the definite integrals.

13. $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$

$u = 2x$

$\frac{du}{2} = dx$

$\int_0^{\pi} \sin(u) \left(\frac{du}{2}\right)$

$\frac{1}{2}[-\cos u] \Big|_0^{\pi}$

$\frac{1}{2}[-(-1) - -1]$

$\frac{1}{2}[1+1]$

$\boxed{1}$

14. $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{1}{1+9t^2} dt$ ← $\tan^{-1} t$

$\int_{-1/3}^{1/3} \frac{1}{1+(3t)^2} dt$

$u = 3t$

$\frac{du}{3} = dt$

$\int_{-1}^{1} \frac{1}{1+u^2} \left(\frac{du}{3}\right)$

$\frac{1}{3}[\tan^{-1} u] \Big|_{-1}^{1}$

$\frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(-1)]$

$\frac{1}{3}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$

$\boxed{\frac{\pi}{6}}$

15. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

$u = 2x + 1$

$\frac{du}{2} = dx$

$\int_1^9 \frac{1}{\sqrt{u}} \left(\frac{du}{2}\right)$

$\frac{1}{2} \int_1^9 u^{-1/2} du$

$\frac{1}{2} u^{1/2} \cdot 2 \Big|_1^9$

$\sqrt{u} \Big|_1^9$

$\sqrt{9} - \sqrt{1}$

$\boxed{2}$

16. $\int_{-\frac{\pi}{4}}^{0} \tan x \sec^2 x \, dx$

$u = \tan x$

$\frac{du}{\sec^2 x} = dx$

$\int_{-1}^{0} u \sec^2 x \left(\frac{du}{\sec^2 x}\right)$

$\int_{-1}^{0} u \, du$

$\frac{u^2}{2} \Big|_{-1}^{0}$

$(0) - (\frac{1}{2})$

$\boxed{-\frac{1}{2}}$

17. $\int_0^{\frac{\pi}{8}} \sec(2x)\tan(2x)\, dx$

$u = 2x$

$\frac{du}{2} = dx$

$\int_0^{\pi/4} \sec(u)\tan(u) \frac{du}{2}$

$\frac{1}{2}\int_0^{\pi/4} \frac{1}{\cos u} \cdot \frac{\sin u}{\cos u} du$

$w = \cos u$

$\frac{dw}{-\sin u} = du$

$\frac{1}{2}\int_1^{\sqrt{2}/2} \frac{1}{w^2} \sin u \left(\frac{dw}{-\sin u}\right)$

$-\frac{1}{2}\int_1^{\sqrt{2}/2} w^{-2} dw$

$-\frac{1}{2} \frac{w^{-1}}{-1} \Big|_1^{\sqrt{2}/2}$

$\frac{1}{2}\left[\frac{1}{\sqrt{2}/2} - 1\right] = \boxed{\frac{\sqrt{2}}{2} - \frac{1}{2}}$

18. $\int_1^e \frac{\ln x}{x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$x\,du = dx$

$\int_0^1 \frac{u}{x}(x\,du)$

$\int_0^1 u\,du$

$\frac{u^2}{2}\Big|_0^1$

$\boxed{\frac{1}{2}}$

19. $\int_0^1 \frac{x^2+2x}{\sqrt[3]{x^3+3x^2+4}} dx$

$u = x^3 + 3x^2 + 4$

$du = (3x^2 + 6x)\,dx$

$\frac{du}{3(x^2+2x)} = dx$

$\frac{1}{3}\int_4^8 u^{-1/3} du$

$\frac{1}{3}\left[u^{2/3} \cdot \frac{3}{2}\right]\Big|_4^8$

$\frac{1}{2}\left[8^{2/3} - 4^{2/3}\right]$

$\frac{1}{2}\left[4 - \sqrt[3]{16}\right]$

$\boxed{2 - \frac{1}{2}\sqrt[3]{16}}$

20. $\int_0^{\pi} (2\sin x + \sin 2x) dx$

$u = 2x$

$\frac{du}{2} = dx$

$2\int_0^{\pi} \sin x \, dx + \int_0^{\pi} \sin(2x)\, dx$

$2[-\cos x]\Big|_0^{\pi} + \frac{1}{2}\int_0^{2\pi} \sin u \, du$

$2[-(-1) - -1] + \frac{1}{2}[-\cos u]\Big|_0^{2\pi}$

$4 + \frac{1}{2}[-(1) - -(1)]$

$\boxed{4}$

6.9 Integrating Using Substitution

Test Prep

21. If $\int_0^k \frac{x}{x^2+6} dx = \frac{1}{2}\ln 6$, where $k > 0$, then $k =$

$u = x^2 + 6$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\frac{1}{2}\int_6^{k^2+6} \frac{1}{u} du$

$\frac{1}{2} \ln|u| \Big|_6^{k^2+6}$

$\frac{1}{2}\left[\ln(k^2+6) - \ln 6\right] = \frac{1}{2}\ln 6$

$\ln(k^2+6) - \ln 6 = \ln 6$

$\ln(k^2+6) = 2\ln 6$

$\ln(k^2+6) = \ln 36$

$k^2 + 6 = 36$

(A) $\sqrt{6}$ (B) $\sqrt{30}$ (C) $\ln 6$ (D) $\frac{1}{2}\tan^{-1}(x)$ (E) $\frac{1}{2}\tan x$

22. The function f is continuous and $\int_4^{19} f(u)\,du = 10$. What is the value of $\int_1^4 [x \cdot f(x^2+3)]\,dx$

$u = x^2+3$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int_4^{19} [x \cdot f(u)]\left(\frac{du}{2x}\right)$

$\frac{1}{2}\int_4^{19} f(u)\,du = \frac{1}{2} \cdot 10$

(A) $\frac{5}{2}$ (B) 5 (C) 10 (D) 20 (E) 40

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23. $\int \frac{1}{\sqrt{16-x^2}}\,dx$

$\sqrt{16} = 4$, so $\div$ top and bottom by "4"

$\int \frac{\frac{1}{4}}{\sqrt{\frac{16}{16} - \frac{x^2}{16}}}\,dx \rightarrow \int \frac{1}{4\sqrt{1-\left(\frac{x}{4}\right)^2}}\,dx$

$u = \frac{x}{4}$

$du = \frac{1}{4}\,dx$

$4\,du = dx$

$\frac{1}{4}\int \frac{1}{\sqrt{1-u^2}}(4\,du)$

$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C$

(A) $\ln\left(\sqrt{16-x^2}\right) + C$ (B) $\ln\frac{\left(\sqrt{16-x^2}\right)}{-2x} + C$ (C) $\frac{1}{4}\sin^{-1}\left(\frac{x}{4}\right) + C$

(D) $4\sin^{-1}\left(\frac{x}{4}\right) + C$ (E) $\sin^{-1}\left(\frac{x}{4}\right) + C$