7.1 Modeling with Differential Equations

Calculus

Solutions

Practice

Write a differential equation that describes each relationship. If necessary, use k as the constant of proportionality.

1. The rate of change of F with respect to s is inversely proportional to x.

$$\frac{dF}{ds} = \frac{k}{x}$$

2. The rate of change of R with respect to t is proportional to the product of S and T.

$$\frac{dR}{dt} = kST$$

3. The rate of change of R with respect to x is proportional to m and inversely proportional to the square of d.

$$\frac{dR}{dx} = k\frac{m}{d^2}$$

4. Kinetic energy, E, changes with respect to t at a rate that is proportional to the square of the velocity, v, and inversely proportional to the mass, m.

$$\frac{dE}{dt} = k\frac{v^2}{m}$$

5. The rate of change of the volume, $V(t)$, of a right circular cone with respect to time (in seconds) is increasing at a rate proportional to the product of the square of its radius, r, and its height, h. Find the differential equation if the cone has a height of 8 inches, radius 3 inches, and the volume is changing by 2 cubic inches per second.

$$\frac{dV}{dt} = kr^2 \cdot h$$

$$2 = k(3)^2(8)$$

$$0.0277 \approx k$$

$$\boxed{\frac{dV}{dt} = 0.0277\, r^2 h}$$

6. The maximum safe load, S, for a horizontal beam of fixed width changes depending on the length, l of the beam. S decreases (with respect to l) proportional to the product of its width, w, the square of its height, h, and inversely as its length, l.

$$\frac{dS}{dl} = k\frac{wh^2}{l}$$

7. Mr. Kelly is running down his street. His position is given by the function $p(t)$, where t is measured in seconds since the start of his run. His acceleration is proportional to the square root of the time since the start of his run. His acceleration is 0.0277 km/sec/sec at 3 seconds.

$$\frac{d^2p}{dt^2} = k\sqrt{t}$$

$$0.0277 = k\sqrt{3}$$

$$0.016 \approx k$$

$$\boxed{\frac{d^2p}{dt^2} = 0.016\sqrt{t}}$$

8. The amount of time, T, that it takes fruit to ripen changes depending on the earth's latitude, L. The rate of change of T, with respect to the latitude, is inversely proportional to the square of L.

$$\frac{dT}{dL} = \frac{k}{L^2}$$

9. Mr. Brust is outside singing one evening, and the neighborhood dogs chime in. The longer he sings, the more difficult it is for him to maintain a solid tone. His voice change can be modeled by the rate of change of frequency, F, with respect to time that is inversely proportional to D, the decibel level of his voice. If the frequency is changing by 3 vibrations per second when he is projecting at 80 decibels, what is a differential equation that describes this relationship?

$$\frac{dF}{dt} = \frac{k}{D}$$

$$3 = \frac{k}{80}$$

$$240 = k$$

$$\boxed{\frac{dF}{dt} = \frac{240}{D}}$$

10. The height of a rocket is given by the function $h(t)$, where t is measured in seconds since the launch and h is measured in meters. The acceleration is proportional to the square root of the time since the start of the launch. At 21 seconds, the acceleration is 5 meters per second per second.

$$\frac{d^2h}{dt^2} = k\sqrt{t}$$

$$5 = k\sqrt{21}$$

$$1.091 \approx k$$

$$\boxed{\frac{d^2h}{dt^2} = 1.091\sqrt{t}}$$

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Test Prep

11. If $s(t)$ is a baby's shoe size at time t, which of the following differential equations describes linear growth in the size of the baby's shoes.

derivative of a linear function is a constant.

(A) $\frac{ds}{dt} = 2s$ (B) $\frac{ds}{dt} = s^2$ (C) $\frac{ds}{dt} = 2$ (D) $\frac{ds}{dt} = 2t$ (E) $\frac{ds}{dt} = t^2$

12. The rate at which a quantity B of a certain substance decays is proportional to the amount of the substance present at a given time. Which of the following is a differential equation that could describe this relationship.

(A) $\frac{dB}{dt} = -2.4t^2$ (B) $\frac{dB}{dt} = 0.04t^2$ (C) $\frac{dB}{dt} = -0.32B$ (D) $\frac{dB}{dt} = 2.1B$