7.2 Verifying Solutions

Calculus

Solutions

Practice

For each differential equation, find the particular solution that passes through the given point.

1. $\frac{dy}{dx} = 4x + 2;\ (-1, 3)$

$y = \int 4x + 2\ dx$

$y = \frac{4x^2}{2} + 2x + C$

$3 = 2(-1)^2 + 2(-1) + C$

$3 = 0 + C$

$3 = C$

$\boxed{y = 2x^2 + 2x + 3}$

2. $\frac{dy}{dx} = \frac{3}{2-x} + 6x^2;\ \ (1, 1)$

$u = 2 - x$

$-du = dx$

$y = \int \frac{3}{u}(-du) + \int 6x^2 dx$

$y = -3\ln|2-x| + \frac{6x^3}{3} + C$

$1 = -3\ln|2-1| + 2 + C$

$1 = 0 + 2 + C$

$-1 = C$

$\boxed{y = -3\ln|2-x| + 2x^3 - 1}$

3. $\frac{dy}{dx} = 8\cos(4x);\ \left(\frac{\pi}{8}, -2\right)$

$u = 4x$

$\frac{du}{4} = dx$

$y = 8\int \cos(u)\left(\frac{dx}{4}\right)$

$y = 2\sin(4x) + C$

$-2 = 2\sin\left(\frac{\pi}{2}\right) + C$

$-2 = 2 + C$

$-4 = C$

$\boxed{y = 2\sin(4x) - 4}$

4. $\frac{dy}{dx} = 9e^{3x} - 1;\ (0, 7)$

$u = 3x$

$\frac{du}{3} = dx$

$y = 9\int e^u \frac{du}{3} - \int 1\, dx$

$y = 3e^{3x} - x + C$

$7 = 3e^0 - 0 + C$

$7 = 3 + C$

$4 = C$

$\boxed{y = 3e^{3x} - x + 4}$

5. $\frac{d^2y}{dx^2} = \frac{1}{(2-x)^2} + 1$ and $y'(3) = 6$ and $y(1) = 4$

$u = 2 - x$

$\frac{du}{-1} = dx$

$\frac{dy}{dx} = \int u^{-2}\left(\frac{du}{-1}\right) + \int 1\, dx$

$\frac{dy}{dx} = -\frac{u^{-1}}{-1} + x + C$

$\frac{dy}{dx} = \frac{1}{2-x} + x + C$

$6 = \frac{1}{2-3} + 3 + C$

$6 = -1 + 3 + C$

$4 = C$

$\frac{dy}{dx} = \frac{1}{2-x} + x + 4$

$y = \int \frac{1}{u}\left(\frac{du}{-1}\right) + \int (x+4)\, dx$

$y = -\ln|2-x| + \frac{1}{2}x^2 + 4x + C$

$4 = -\ln|2-1| + \frac{1}{2} + 4 + C$

$4 = 4.5 + C$

$-0.5 = C$

$\boxed{y = -\ln|2-x| + \frac{1}{2}x^2 + 4x - \frac{1}{2}}$

6. $\frac{d^2y}{dx^2} = e^{2x} - x$ and $y'(0) = \frac{3}{2}$ and $y(0) = \frac{3}{4}$

$u = 2x$

$\frac{du}{2} = dx$

$\frac{dy}{dx} = \int e^u\left(\frac{du}{2}\right) - \int x\, dx$

$\frac{dy}{dx} = \frac{1}{2}e^{2x} - \frac{1}{2}x^2 + C$

$\frac{3}{2} = \frac{1}{2}e^0 - 0 + C$

$\frac{3}{2} = \frac{1}{2} + C$

$1 = C$

$\frac{dy}{dx} = \frac{1}{2}e^{2x} - \frac{1}{2}x^2 + 1$

$y = \int \frac{1}{2}e^u\left(\frac{du}{2}\right) - \frac{1}{2}\int x^2 dx + \int 1\, dx$

$y = \frac{1}{4}e^{2x} - \frac{1}{6}x^3 + x + C$

$\frac{3}{4} = \frac{1}{4}e^0 - 0 + 0 + C$

$\frac{2}{4} = C$

$\boxed{y = \frac{1}{4}e^{2x} - \frac{1}{6}x^3 + x + \frac{1}{2}}$

Find the value of k of each equation that would be a solution to the given differential equation.

7. $y = 3ke^{2x} + \cos(4x)$

Diff Eq: $\frac{y''}{2} + 8y = 15e^{2x}$

$y' = 6ke^{2x} - 4\sin(4x)$

$y'' = 12ke^{2x} - 16\cos(4x)$

$\frac{12ke^{2x} - 16\cos(4x)}{2} + 8[3ke^{2x} + \cos(4x)] = 15e^{2x}$

$6ke^{2x} - 8\cos(4x) + 24ke^{2x} + 8\cos(4x) = 15e^{2x}$

$\frac{30ke^{2x}}{30e^{2x}} = \frac{15e^{2x}}{30e^{2x}}$

$\boxed{k = 1/2}$

8. $y = k\sin(-x) + 2\cos(3x)$

Diff Eq: $2y'' + 18y = 32\sin(-x)$

$y' = -k\cos(-x) - 6\sin(3x)$

$y'' = -k\sin(-x) - 18\cos(3x)$

$2[-k\sin(-x) - 18\cos(3x)] + 18[k\sin(-x) + 2\cos(3x)] = 32\sin(-x)$

$-2k\sin(-x) - 36\cos(3x) + 18k\sin(-x) + 36\cos(3x) = 32\sin(-x)$

$16k\sin(-x) = 32\sin(-x)$

$\boxed{k = 2}$

9. $y = e^{-3x} + ke^{4x}$

Diff Eq: $3y' + y'' = -14e^{4x}$

$y' = -3e^{-3x} + 4ke^{4x}$

$y'' = 9e^{-3x} + 16ke^{4x}$

$3[-3e^{-3x} + 4ke^{4x}] + [9e^{-3x} + 16ke^{4x}] = -14e^{4x}$

$-9e^{-3x} + 12ke^{4x} + 9e^{-3x} + 16ke^{4x} = -14e^{4x}$

$28ke^{4x} = -14e^{4x}$

$\boxed{k = -1/2}$

10. $y = e^{3x} + ke^{-2x}$

Diff Eq: $y'' - 2y' - 3y = 4e^{-2x}$

$y' = 3e^{3x} - 2ke^{-2x}$

$y'' = 9e^{3x} + 4ke^{-2x}$

$[9e^{3x} + 4ke^{-2x}] - 2[3e^{3x} - 2ke^{-2x}] - 3[e^{3x} + ke^{-2x}] = 4e^{-2x}$

$9e^{3x} + 4ke^{-2x} - 6e^{3x} + 4ke^{-2x} - 3e^{3x} - 3ke^{-2x} = 4e^{-2x}$

$5ke^{-2x} = 4e^{-2x}$

$\boxed{k = 4/5}$

11. Of the following, which are solutions to the differential equation $y'' - 5y' + 4y = 0$
    I. $y = 5\cos(2x)$
    II. $y = 2e^x$
    III. $y = Ce^{4x}$, where C is a constant.

I. $y = 5\cos(2x)$
$y' = -10\sin(2x)$
$y'' = -20\cos(2x)$
$\rightarrow -20\cos(2x) + 50\sin(2x) + 20\cos(2x) = 50\sin(2x)$ ✗

II. $y = 2e^x$
$y' = 2e^x$
$y'' = 2e^x$
$\rightarrow 2e^x - 10e^x + 8e^x = 0$ ✓

III. $y = Ce^{4x}$
$y' = 4Ce^{4x}$
$y'' = 16Ce^{4x}$
$\rightarrow 16Ce^{4x} - 20Ce^{4x} + 4Ce^{4x} = 0$ ✓

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

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12. Consider the differential equation $\frac{dy}{dx} = (y-4)^3 \sin\left(\frac{\pi x}{2}\right)$. There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

$y = 4$. A horizontal line exists only if the slope (derivative) is zero. This differential equation will equal zero when $y = 4$.