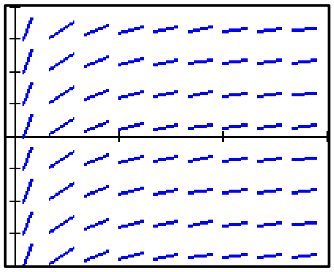
7.4 Reasoning Using Slope Fields

Calculus

Practice

The slope field from a certain differential equation is shown for each problem. The multiple choice answers are either differential equations OR a specific solution to that differential equation.

1.



(A) $y = \ln x$

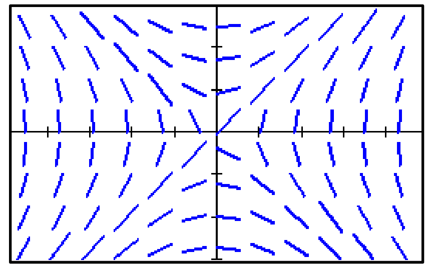
(B) $y = e^{x}$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = x^2$

2.



(A) $\frac{dy}{dx} = x + y$

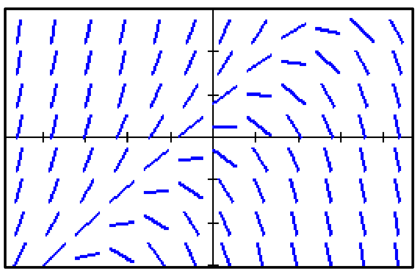
(B) $\frac{dy}{dx} = \frac{x}{y}$

(C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = (x-1)y$

(E) $\frac{dy}{dx} = x(y-1)$

3.



(A) $\frac{dy}{dx} = y - x$

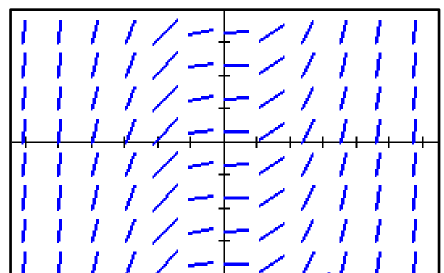
(B) $\frac{dy}{dx} = -\frac{x}{y}$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = y(x-1)$

(E) $\frac{dy}{dx} = x(y-1)$

4.



(A) $y = \sin x$

(B) $y = \cos x$

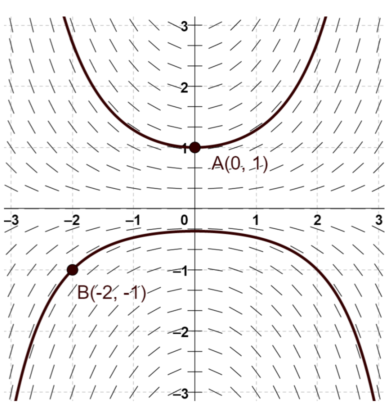
(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{4}x^4$

For each slope field, plot and label the points A and B and sketch the particular solution that passes through each of those points. (Two separate solutions for each slope field.)

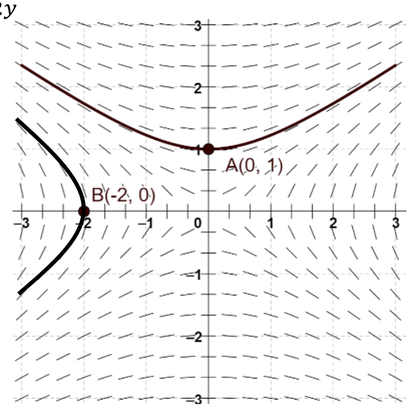
5. $\frac{dy}{dx} = \frac{xy}{2}$



Point A: $(0, 1)$

Point B: $(-2, -1)$

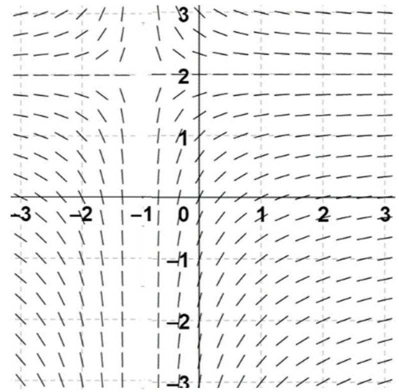
6. $\frac{dy}{dx} = \frac{x}{2y}$



Point A: $(0, 1)$

Point B: $(-2, 0)$

7. The slope field for a certain differential equation is shown. Which of the following could be a solution to the differential equation with initial condition $y(2) = 0$?



(A) $y = \frac{2x-4}{x+1}$

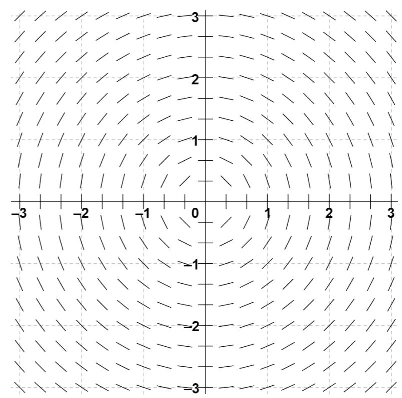
(B) $y = \frac{4}{x+1} - 2$

(C) $y = \ln|1-x|$

(D) $y = \frac{3x^2}{x+1} - 6$

A

8. The slope field for a certain differential equation is shown. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?



(A) $y = \frac{x}{y} + 1$

(B) $y = -\frac{x}{y} + 1$

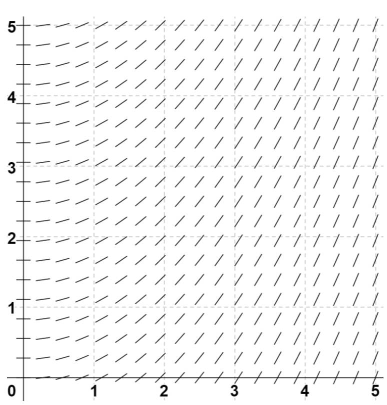
(C) $x^2 + (y+1)^2 = 4$

(D) $x^2 + y^2 = 1$

D

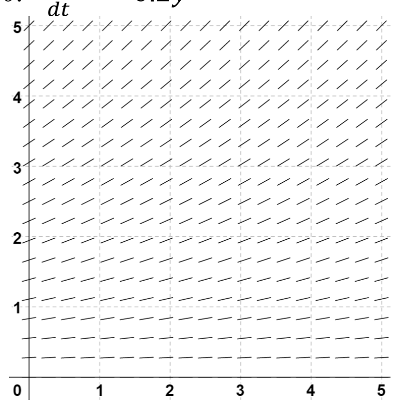
For each problem below a slope field and a differential equation are given. Explain why the slope field CANNOT represent the differential equation.

9. $\frac{dy}{dt} = 0.5y$



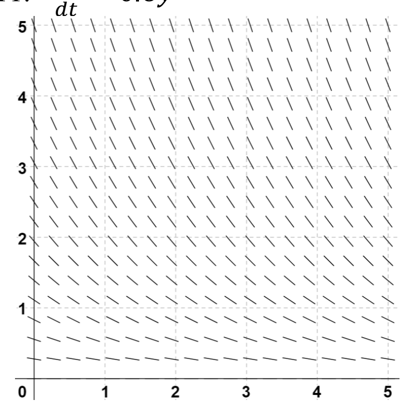
One possible answer: When $y = 0, \frac{dy}{dt} = 0$. However, in the slope field, the slopes of the line segments for $y = 0$ are nonzero.

10. $\frac{dy}{dt} = -0.2y$



$\frac{dy}{dx} < 0$ when $y > 0$, but the slope field shows line segments with positive slope.

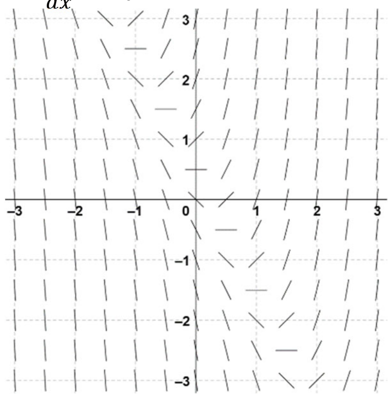
11. $\frac{dy}{dt} = 0.6y$



$\frac{dy}{dx} > 0$ when $y > 0$, but the slope field shows line segments with negative slope.

Consider the differential equation and its slope field. Describe all points in the xy-plane that match the given condition.

12. $\frac{dy}{dx} = 2y + 4x - 1$



When is $\frac{dy}{dx}$ is positive?

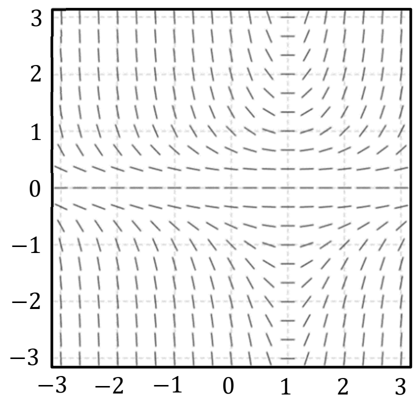
$2y + 4x - 1 > 0$

$2y > -4x + 1$

$y > -2x + \frac{1}{2}$

All points that make $y > -2x + \frac{1}{2}$ true.

13. $\frac{dy}{dx} = y^2(x - 1)$



When are the slopes nonnegative?

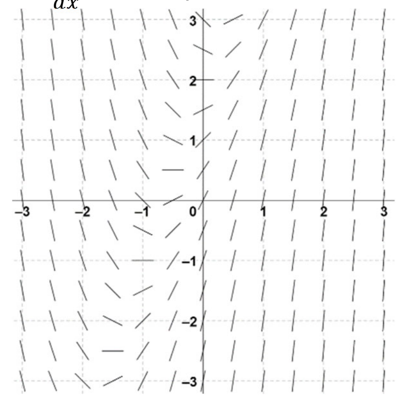
$y^2(x-1) \geq 0$

$y^2 \geq 0 \qquad x - 1 \geq 0$

Always positive $\qquad x \geq 1$

If $x \geq 1$, the slope is nonnegative.

14. $\frac{dy}{dx} = 3x - y + 2$



When does $\frac{dy}{dx} = 1$?

$3x - y + 2 = 1$

$-y = -3x - 1$

$y = 3x + 1$

All points on the line $y = 3x + 1$ make $\frac{dy}{dx} = 1$.

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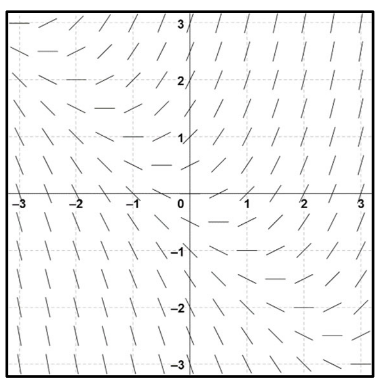
Test Prep

15.



The slope field for a certain differential equation is shown above. Which of the following statements about a solution $y = f(x)$ to the differential equation must be false?

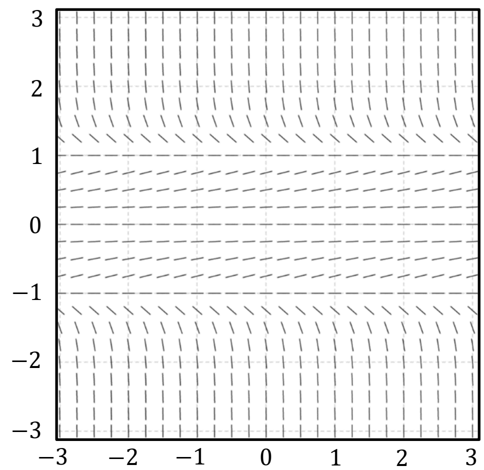
(A) The graph of the particular solution that satisfies $f(2) = -2$ has a relative minimum at $x = 2$.

(B) The graph of the particular solution that satisfies $f(-1) = -1$ is concave up on the interval $-2 < x < 1$.

(C) The graph of the particular solution that satisfies $f(1) = -2$ is linear.

(D) The graph of the particular solution that satisfies $f(-1) = 2$ is concave up on the interval $-3 < x < 3$.

16.



Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2(1 - y^2)$. If $y = f(x)$ is the solution to the differential equation with initial condition $f(1) = 2$, then $\lim_{x\to\infty} f(x)$ is

(A) $-\infty$　　(B) -1　　(C) 0　　(D) 1　　(E) ∞

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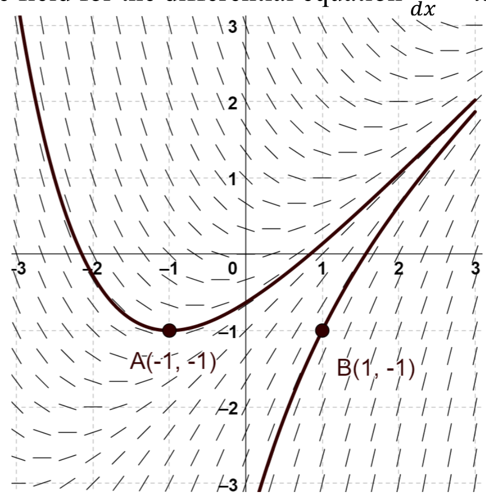
17. The figure below shows the slope field for the differential equation $\frac{dy}{dx} = x - y$



a. Sketch the graph of a particular solution that contains $(-1, -1)$. Label this point as Point A.

b. Sketch the graph of a particular solution that contains $(1, -1)$. Label this point as Point B.

c. State a point where $\frac{dy}{dx} = 0$. Find $\frac{d^2y}{dx^2}$ and use it to verify if your point is a max or min.

Answers will vary. One example is the point $(1, 1)$. Because $\frac{d^2y}{dx^2} > 0$ because the slope field shows a concave up graph. Because $\frac{dy}{dx} = 0$ as well, this point represents a minimum.