Calculus

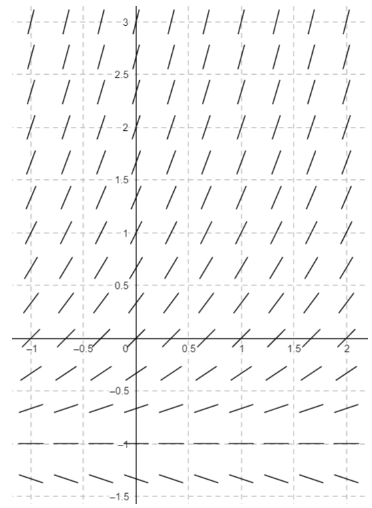
7.5 Euler's Method

Notes

Write your questions and thoughts here!

In lesson 7.6, we will show how to find a solution $y = f(x)$ to a differential equation. In this lesson, we are going to APPROXIMATE a solution to a differential equation. This approximation method is called **Euler's method**.

1. $\frac{dy}{dx} = 1 + y$ and $y(0) = 0$. $\Delta x = 0.5$. Using Euler's Method, show an approximation to the solution curve $y = f(x)$.



Step 1: Construct a tangent line at $(0, 0)$ for $0 \leq x \leq 0.5$.

Starting point was $(0, 0)$. New point to work with is

Step 2: Construct a tangent line at for $0.5 \leq x \leq 1$.

Starting point was New point to work with is

Step 3: Construct a tangent line at for $1 \leq x \leq 1.5$.

Starting point was New point to work with is

Write your questions and thoughts here!

Here is a way Euler Method questions often appear on the AP Exam.

2. $\frac{dy}{dx} = 2x$ and let $f(x) = y$ be a solution to this differential equation. If $f(1) = 3$, what is the approximation to $f(2)$ obtained by using Euler's method with 5 steps of equal size?

First, find the step size. $\Delta x =$

$$y - y_1 = m(x - x_1)$$

$$y = y_1 + m(x - x_1)$$

x	y	y'	New y

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Calculus

Practice

1. The table below gives the values of f', the derivative of f. If $f(1) = 2$, what is the approximation to $f(2.5)$ obtained by using Euler's method with 3 steps of equal size?

x	1	1.5	2.0	2.5
$f'(x)$	0.3	0.7	1.2	1.8

2. The table below gives the values of f', the derivative of f. If $f(2) = 3$, what is the approximation to $f(2.6)$ obtained by using Euler's method with 2 steps of equal size?

x	2	2.3	2.6
$f'(x)$	-0.5	-0.3	-0.1

3. The table below gives the values of f', the derivative of f. If $f(3) = 5$, what is the approximation to $f(4.0)$ obtained by using Euler's method with 2 steps of equal size?

x	3	3.25	3.5	3.75	4.0	4.25
$f'(x)$	0.1	0.3	0.5	0.7	0.9	1.1

4. The table below gives the values of f', the derivative of f. If $f(1.5) = 4$, what is the approximation to $f(1)$ obtained by using Euler's method with 2 steps of equal size?

x	1	1.25	1.5	1.75	2.0
$f'(x)$	0.3	0.4	0.6	0.9	1.3

5. Let $h(x) = \int_1^x \sqrt{1+t^2}\,dt$. Use Euler's method, starting at $x = 1$ with 2 steps of equal size, to approximate $h(3)$.

6. Let $h(x) = \int_0^x \sqrt{1 + 3t^2}\, dt$. Use Euler's method, starting at $x = 0$ with 3 steps of equal size, to approximate $h(3)$.

7. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x - y$ with initial condition $f(1) = 0$. What is the approximation for $f(1.3)$ obtained using Euler's method with 3 steps of equal length, starting at $x = 1$?

8. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ with initial condition $f(0) = 1$. What is the approximation for $f(.3)$ obtained using Euler's method with 3 steps of equal length, starting at $x = 0$?

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y$ with initial condition $f(0) = 1$. What is the approximation for $f(.5)$ obtained using Euler's method with a step size of $\Delta x = 0.1$, starting at $x = 0$?

10. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(.8)$ obtained using Euler's method with 4 steps of equal length, starting at $x = 0$?

No Test Prep section for this lesson as the practice problems are similar to what will be on the AP Exam.