7.5 Euler's Method

Calculus

Practice

1. The table below gives the values of f', the derivative of f. If $f(1) = 2$, what is the approximation to $f(2.5)$ obtained by using Euler's method with 3 steps of equal size?

x	1	1.5	2.0	2.5
$f'(x)$	0.3	0.7	1.2	1.8

x	y	y'	new y
1	2	0.3	$2+0.3(0.5) = 2.15$
1.5	2.15	0.7	$2.15+0.7(0.5) = 2.5$
2	2.5	1.2	$2.5+1.2(0.5) = 3.1$
2.5	3.1		

$y = y_1 + y' \cdot (\Delta x)$

$\Delta x = 0.5$

$f(2.5) \approx 3.1$

2. The table below gives the values of f', the derivative of f. If $f(2) = 3$, what is the approximation to $f(2.6)$ obtained by using Euler's method with 2 steps of equal size?

x	2	2.3	2.6
$f'(x)$	−0.5	−0.3	−0.1

x	y	y′	new y
2	3	−0.5	3+−0.5(0.3)=2.85
2.3	2.85	−0.3	2.85+−0.3(0.3)=2.76
2.6	2.76		

$y = y_1 + y'\cdot(\Delta x)$

$\Delta x = 0.3$

$f(2.6) \simeq 2.76$

3. The table below gives the values of f', the derivative of f. If $f(3) = 5$, what is the approximation to $f(4.0)$ obtained by using Euler's method with 2 steps of equal size?

x	3	3.25	3.5	3.75	4.0	4.25
$f'(x)$	0.1	0.3	0.5	0.7	0.9	1.1

x	y	y′	new y
3	5	0.1	5+0.1(0.5)=5.05
3.5	5.05	0.5	5.05+0.5(0.5)=5.3
4	5.3		

$y = y_1 + y'\cdot(\Delta x)$

$\Delta x = 0.5$

$f(4) \simeq 5.3$

4. The table below gives the values of f', the derivative of f. If $f(1.5) = 4$, what is the approximation to $f(1)$ obtained by using Euler's method with 2 steps of equal size?

x	1	1.25	1.5	1.75	2.0
$f'(x)$	0.3	0.4	0.6	0.9	1.3

x	y	y′	new y
1.5	4	0.6	4+0.6(−0.25)=3.85
1.25	3.85	0.4	3.85+0.4(−0.25)=3.75
1	3.75		

$y = y_1 + y'\cdot(\Delta x)$ Negative!!

$\Delta x = -0.25$

$f(1) \simeq 3.75$

5. Let $h(x) = \int_1^x \sqrt{1+t^2}\,dt$. Use Euler's method, starting at $x = 1$ with 2 steps of equal size, to approximate $h(3)$.

$h'(x) = \sqrt{1+x^2}$

x	h(x)	h′(x)	new h(x)
1	0	$\sqrt{1+1^2}=\sqrt{2}$	$0+\sqrt{2}(1)=\sqrt{2}$
2	$\sqrt{2}$	$\sqrt{1+2^2}=\sqrt{5}$	$\sqrt{2}+\sqrt{5}(1)=\sqrt{2}+\sqrt{5}$
3	$\sqrt{2}+\sqrt{5}$		

$y = y_1 + y'\cdot(\Delta x)$

$\Delta x = 1$

$h(3) \simeq \sqrt{2} + \sqrt{5}$

6. Let $h(x) = \int_0^x \sqrt{1+3t^2}\,dt$. Use Euler's method, starting at $x = 0$ with 3 steps of equal size, to approximate $h(3)$.

$h'(x) = \sqrt{1+3x^2}$

$y = y_1 + y' \cdot (\Delta x)$ ← $\Delta x = 1$

x	h(x)	h′(x)	new h(x)
0	0	1	$0 + 1(1) = 1$
1	1	$\sqrt{1+3} = 2$	$1 + 2(1) = 3$
2	3	$\sqrt{1+12} = \sqrt{13}$	$3 + \sqrt{13}(1)$
3	$3 + \sqrt{13}$		

$h(3) \simeq 3 + \sqrt{13}$

7. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x - y$ with initial condition $f(1) = 0$. What is the approximation for $f(1.3)$ obtained using Euler's method with 3 steps of equal length, starting at $x = 1$?

$y = y_1 + y' \cdot (\Delta x)$ ← $\Delta x = \frac{1.3-1}{3} = 0.1$

x	y	y′	new y
1	0	$2(1) - 0 = 2$	$0 + 2(0.1) = 0.2$
1.1	0.2	$2(1.1) - 0.2 = 2$	$0.2 + 2(0.1) = 0.4$
1.2	0.4	$2(1.2) - 0.4 = 2$	$0.4 + 2(0.1) = 0.6$
1.3	0.6		

$f(1.3) \simeq 0.6$

8. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ with initial condition $f(0) = 1$. What is the approximation for $f(0.3)$ obtained using Euler's method with 3 steps of equal length, starting at $x = 0$?

$y = y_1 + y' \cdot (\Delta x)$ ← $\Delta x = \frac{0.3-0}{3} = 0.1$

x	y	y′	new y
0	1	$-\frac{0}{1} = 0$	$1 + 0(0.1) = 1$
0.1	1	$-\frac{0.1}{1} = -0.1$	$1 - 0.1(0.1) = 0.99$
0.2	0.99	$-\frac{0.2}{0.99} = -0.202$	$0.99 - 0.202(0.1) = 0.9698$
0.3	0.9698		

$f(0.3) \simeq 0.9698$

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.5)$ obtained using Euler's method with a step size of $\Delta x = 0.1$, starting at $x = 0$?

$y = y_1 + y' \cdot (\Delta x)$ ← 0.1

x	y	y′	new y
0	1	1	$1 + 1(0.1) = 1.1$
0.1	1.1	1.1	$1.1 + 1.1(0.1) = 1.21$
0.2	1.21	1.21	$1.21 + 1.21(0.1) = 1.331$
0.3	1.331	1.331	$1.331 + 1.331(0.1) = 1.4641$
0.4	1.4641	1.4641	$1.4641 + 1.4641(0.1) = 1.61051$
0.5	1.61051		

$f(0.5) \simeq 1.6105$

10. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.8)$ obtained using Euler's method with 4 steps of equal length, starting at $x = 0$?

$y = y_1 + y' \cdot (\Delta x)$ ← $\Delta x = \frac{0.8-0}{4} = 0.2$

x	y	y′	new y
0	1	$0 + 1 = 1$	$1 + 1(0.2) = 1.2$
0.2	1.2	1.4	$1.2 + 1.4(0.2) = 1.48$
0.4	1.48	1.88	$1.48 + 1.88(0.2) = 1.856$
0.6	1.856	2.456	$1.856 + 2.456(0.2) = 2.3472$
0.8	2.3472		

$f(0.8) \simeq 2.347$

No Test Prep section for this lesson as the practice problems are similar to what will be on the AP Exam.