

7.6 Separation of Variables (General Solutions)

Calculus

Solutions

Practice

Find the general solution of each differential equation.

1. $\frac{dy}{dx} = \frac{3x^2}{y}$

$$y \, dy = 3x^2 \, dx$$

$$\frac{1}{2}y^2 = x^3 + C$$

$$y^2 = 2x^3 + C$$

$$y = \pm \sqrt{2x^3 + C}$$

2. $\frac{dy}{dx} = 8x^2y$

$$\frac{1}{y} \, dy = 8x^2 \, dx$$

$$\ln|y| = \frac{8}{3}x^3 + C$$

$$e^{\ln|y|} = e^{\frac{8}{3}x^3 + C}$$

$$y = Ce^{\frac{8}{3}x^3}$$

3. $\frac{dy}{dx} = e^x y^2$

$$y^{-2} \, dy = e^x \, dx$$

$$-y^{-1} = e^x + C$$

$$\frac{1}{y} = -e^x + C$$

$$y = \frac{1}{-e^x + C}$$

4. $\frac{dy}{dx} = -2x(y-3)$

$$\frac{1}{y-3} \, dy = -2x \, dx$$

$$\ln|y-3| = -x^2 + C$$

$$e^{\ln|y-3|} = e^{-x^2 + C}$$

$$y = \frac{C}{e^{x^2}} + 3$$

$$5. \frac{dy}{dx} = y \cos x$$

$$\frac{1}{y} dy = \cos x dx$$
$$\ln|y| = \sin x + C$$
$$e^{\ln|y|} = e^{\sin x + C}$$
$$y = e^{\sin x + C}$$

$$y = Ce^{\sin x}$$

$$6. \frac{dy}{dx} = (y+5)(x+2)$$

$$\frac{1}{y+5} dy = (x+2) dx$$
$$\ln|y+5| = \frac{x^2}{2} + 2x + C$$
$$e^{\ln|y+5|} = e^{\frac{x^2}{2} + 2x + C}$$
$$y+5 = e^{\frac{x^2}{2} + 2x + C}$$

$$y = Ce^{\frac{x^2}{2} + 2x} - 5$$

$$7. \frac{dy}{dx} = e^{x-y}$$

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

$$e^y dy = e^x dx$$

$$e^y = e^x + C$$

$$\ln(e^y) = \ln(e^x + C)$$

$$y = \ln(e^x + C)$$

$$8. \frac{dy}{dx} = \frac{2x}{e^{2y}}$$

$$e^{2y} dy = 2x dx$$

$$\frac{1}{2} e^{2y} = x^2 + C$$

$$e^{2y} = 2x^2 + C$$

$$2y = \ln(2x^2 + C)$$

$$y = \frac{1}{2} \ln(2x^2 + C)$$

$$y = \ln(\sqrt{2x^2 + C})$$

No Test Prep. We will wait for our next lesson when we can use *particular solutions* with separation of variables.