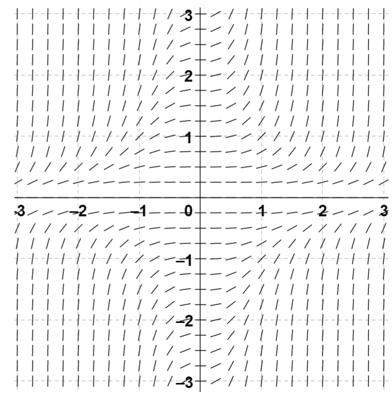
Calculus

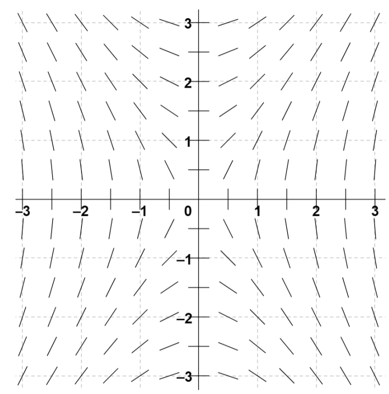
Write your questions and thoughts here!

7.7 Separation of Variables (Particular Solutions) Notes

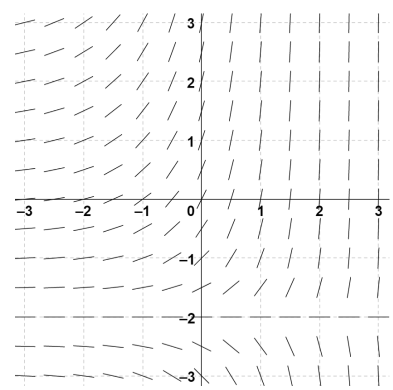
1. Find the solution to the differential equation $\frac{dy}{dx} = (xy)^2$ with initial condition $y(1) = 1$.



2. Find the solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{2x}{y}$ with initial condition $f(2) = 1$.



3. Find the solution to the differential equation $\frac{dy}{dx} = (y + 2)e^x$ with initial condition $y(0) = -1$,



7.7 Separation of Variables (Particular Solutions)

Calculus

Practice

For each differential equation, find the solution that passes through the given initial condition.

1. $\frac{dy}{dx} = y\cos x$ and $y = 4$ when $x = 0$

2. $\frac{dy}{dx} = \frac{6x^2}{y}$ if $y(0) = -2$

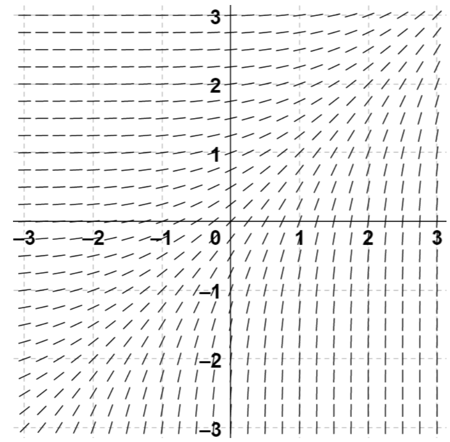
3. $\frac{dy}{dx} = y\sin x$ if $y\left(\frac{\pi}{2}\right) = 2$

4. $\frac{dy}{dx} = \frac{1}{5}(8 - y)$ and $y = 6$ when $x = 0$

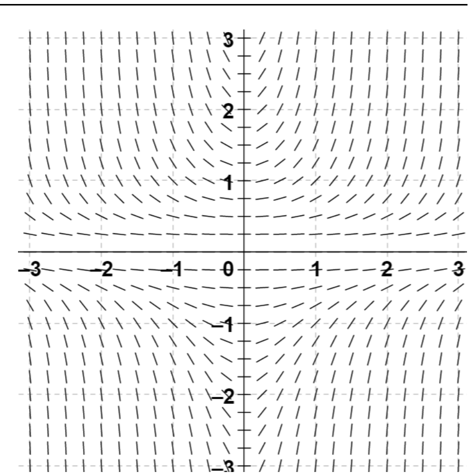
5. $\frac{dy}{dx} = (y + 5)(x + 2)$ when $f(0) = 1$

6. $\frac{dy}{dx} = \frac{e^x}{y}$ if $y(0) = -4$

7. Find the particular solution to $\frac{dy}{dx} = e^{x-y}$ when $f(0) = 2$. Sketch the graph of this particular solution on the slope field provided.



8. Find the particular solution to $\frac{dy}{dx} = xy^2$ if $y = 1$ when $x = 0$. Sketch the graph of this particular solution on the slope field provided.



Test Prep

7.7 Separation of Variables (Particular Solutions)

9. Consider the differential equation $\frac{dy}{dx} = e^y(4x - 1)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through (2,0).

   (a) Write an equation for the line tangent to the graph of f at the point (2,0). Use the tangent line to approximate $f(2.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through (2,0).

10. One of Mr. Kelly's calculus students attempted to solve the differential equation $\frac{dy}{dx} = 2xy$ with initial condition $y = 3$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y} dy = \int 2x\, dx$
Step 2: $\ln|y| = x^2 + C$
Step 3: $|y| = e^{x^2} + C$
Step 4: Since $y = 3$ when $x = 0$, $3 = e^0 + C$.
Step 5: $y = -e^{x^2} + 2$

(A) Step 2 (B) Step 3 (C) Step 4 (D) Step 5 (E) There is no error in the solution.

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11. Consider the differential equation $\frac{dy}{dx} = 6 - 2y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 4$.

a. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 0$. Use the tangent line to approximate $f(0.6)$.

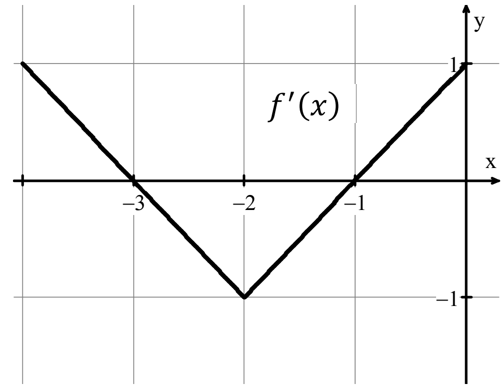
b. Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 4)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(0, 4)$? Give a reason for your answer.

c. Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 4$.

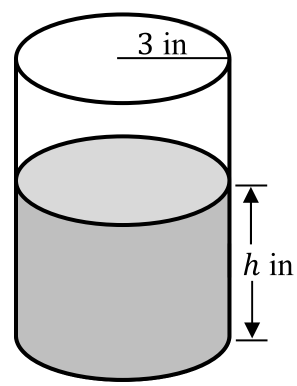
d. For the particular solution $y = f(x)$ found in part c, find $\lim_{x \to \infty} f(x)$.

12. The graph of the derivative of f, f', is shown to the right. The domain of f is the set of all x such that $-4 < x < 0$.

Given that $f(-2) = 0$, find the solution $f(x)$.



13. Mr. Bean's favorite addiction (rhymes with Poctor Depper) is put into a cylindrical container with radius 3 inches, as shown in the figure above. Let h be the depth of the soda in the container, measured in inches, where h is a function of time t, measured in minutes. The volume V of soda in the container is changing at the rate of $-\frac{\pi}{2}\sqrt{h}$ cubic inches per minute throughout the morning. Given that $h = 9$ at the start of 1st period ($t = 0$), solve the differential equation $\frac{dh}{dt}$ for h as a function of t. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



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14. The rate at which a baby koala bear gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bear is first weighed, its weight is 2 pounds. If $B(t)$ is the weight of the bear, in pounds, at time t days after it is first weighed, then $\frac{dB}{dt} = \frac{1}{4}(20 - B)$. Find $y = B(t)$, the particular solution to the differential equation.