## 7.8 Exponential Models with Differential Equations Solutions

Calculus

**Practice** 

Find the particular solution $y = f(t)$ for each differential equation.			
1. $\frac{dy}{dt} = 8y$ and $y = -2$			4. $\frac{dy}{dt} = -7y$ and $y = -4$
when $t = 0$	when $t = 0$	when $t = 0$	when $t = 0$
y=-Σe <sup>st</sup>	y=10e-4t	y=2€ <sup>6t</sup>	y=-4e <sup>-7t</sup>

## For each problem, use your understanding of exponential models and differential equations.

5. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 500 people are infected when the epidemic is first discovered, and 800 people are infected 5 days later, how many people are infected 10 days after the epidemic is first discovered?

Let 
$$P = \# af people infected
P = 500 e^{kt}
800 = 500 e^{k(5)}
 $\frac{3}{5} = e^{5k}$   
 $\ln(\frac{3}{5}) = 5k$   
0.094  $\cong k$   
About 1,280 people$$

6. A population *y* grows according to the equation  $\frac{dy}{dt} = ky$ , where k is a constant and t is measured in years. If the population doubles every 11 years, then what is the value of *k*?

$$\mathcal{J} = Ce^{kt}$$
$$\mathcal{L} = e^{k(n)}$$

$$ln(2) = 11 K$$

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7. A certain animal weighs 30 grams at birth. During the first 4 weeks after the animal's birth, its weight in grams is given by the function W that satisfies the differential equation  $\frac{dW}{dt} = 0.01W$ , where t is measured in days. What is an expression for W(t)?

$$W(t) = 300^{0.01t}$$

9. A population of fruit flies is increasing at an exponential rate. If on the 3<sup>rd</sup> day there were 400 fruit flies, and the 7<sup>th</sup> day there were 600 fruit flies, approximately how many flies were in the original population (day 0)?

$$P = P_{e} e^{kt}$$

$$\frac{400}{e^{3k}} = P_{e} e^{3k}$$

$$\frac{600}{e^{3k}} = P_{e} e^{7k}$$

$$\frac{400}{e^{3k}} = P_{e} e^{7k}$$

$$\frac{600}{e^{7k}} = P_{e} e^{7k}$$

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$$P_{e} = \frac{600}{e^{7(0.10)3}}$$

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$$P_{e} = 295 \text{ flies}$$

$$K \simeq 0.1013$$

11. A radioactive substance has a rate of decay that can be modeled by the equation  $\frac{dy}{dt} = ky$ , where k is a constant and t is measured in years. If the halflife of the radioactive substance is 300 years, then what is the value of k?

$$f = 1 \cdot e^{k(300)}$$

$$ln(2) = 300k$$

$$K \simeq -0.0023$$

8. A population y grows according to the equation  $\frac{dy}{dt} = ky$ , where k is a constant and t is measured in years. If the population doubles every 25 years, then what is the value of k?

$$y = Ce^{kt}$$
  
 $\lambda = e^{k(25)}$   
 $ln(2) = 25 K$   
 $k \simeq 0.0277$ 

10. A petri dish contains 100 bacteria, and the number N of bacteria is increasing according to the equation  $\frac{dN}{dt} = kN$ , where k is a constant and t is measured in hours. At time t = 3, there are 181 bacteria. Based on this information, what is the doubling time for the bacteria?

$$N = N_{0} e^{kt}$$

$$181 = 100 e^{3k}$$

$$1.81 = e^{3k}$$

$$\ln(1.81) = 3k$$

$$K \simeq 0.1977$$

$$N = N_{0} e^{kt}$$

$$2 = e^{0.1977t}$$

$$\ln(2) = 0.1977t$$

$$t \simeq 3.506$$

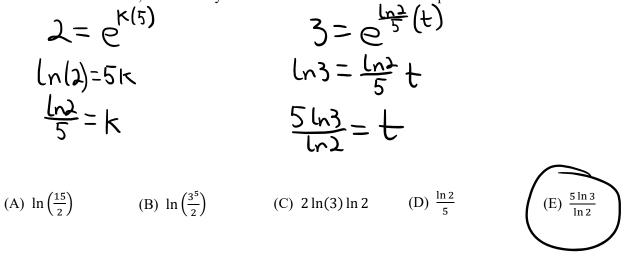
$$haves$$

12. A dose of 100 milligrams of a drug is administered to a patient. The amount of the drug, in milligrams, in the person's bloodstream at time *t*, in hours, is given by A(t). The rate at which the drug leaves the bloodstream can be modeled by the differential equation  $\frac{dA}{dt} = -0.5A$ . Write an expression for A(t).

$$A(t) = |00e^{-0.5t}$$

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13. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in five hours, in how many hours will the number of bacteria triple?



14. A group of tiny organisms (measured in grams) is shrinking a rate modeled by  $\frac{dM}{dt} = -0.7M$ , where the time t is measured in hours. The size of a second group of organisms decreases at a constant rate of 2 grams per hour according to the linear model  $\frac{dN}{dt} = -2$ . If at time t = 0, the first group has size M(0) = 4 and the second colony has size N(0) = 6, at what time will both groups of organisms be the same size?

(A) -0.459

(B) 0.312

(C) 1.292



## **Test Prep**