7.8 Exponential Models with Differential Equations

Calculus

Solutions

Practice

Find the particular solution $y = f(t)$ for each differential equation.

1. $\frac{dy}{dt} = 8y$ and $y = -2$ when $t = 0$

$y = -2e^{8t}$

2. $\frac{dy}{dt} = -4y$ and $y = 10$ when $t = 0$

$y = 10e^{-4t}$

3. $\frac{dy}{dt} = 16y$ and $y = 5$ when $t = 0$

$y = 5e^{16t}$

4. $\frac{dy}{dt} = -7y$ and $y = -4$ when $t = 0$

$y = -4e^{-7t}$

For each problem, use your understanding of exponential models and differential equations.

5. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 500 people are infected when the epidemic is first discovered, and 800 people are infected 5 days later, how many people are infected 10 days after the epidemic is first discovered?

Let P = # of people infected

$P = 500e^{kt}$

$800 = 500e^{k(5)}$

$\frac{8}{5} = e^{5k}$

$\ln(\frac{8}{5}) = 5k$

$0.094 \approx k$

$P = 500e^{.094(10)}$

$P \approx 1279.991$

$\boxed{\text{About 1,280 people}}$

6. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 11 years, then what is the value of k?

$y = Ce^{kt}$

$2 = e^{k(11)}$

$\ln(2) = 11k$

$\boxed{k \approx 0.063}$

7. A certain animal weighs 30 grams at birth. During the first 4 weeks after the animal's birth, its weight in grams is given by the function W that satisfies the differential equation $\frac{dW}{dt} = 0.01W$, where t is measured in days. What is an expression for $W(t)$?

$$\boxed{W(t) = 30e^{0.01t}}$$

8. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 25 years, then what is the value of k?

$$y = Ce^{kt}$$
$$2 = e^{k(25)}$$
$$\ln(2) = 25k$$
$$\boxed{k \approx 0.0277}$$

9. A population of fruit flies is increasing at an exponential rate. If on the 3rd day there were 400 fruit flies, and the 7th day there were 600 fruit flies, approximately how many flies were in the original population (day 0)?

$$P = P_o e^{kt}$$
$$400 = P_o e^{3k} \qquad 600 = P_o e^{7k}$$
$$\frac{400}{e^{3k}} = P_o \qquad \frac{600}{e^{7k}} = P_o$$
$$\frac{400}{e^{3k}} = \frac{600}{e^{7k}} \qquad P_o = \frac{600}{e^{7(0.1013)}}$$
$$e^{4k} = 3/2 \qquad \boxed{P_o \approx 295 \text{ flies}}$$
$$4k = \ln(3/2)$$
$$k \approx 0.1013$$

10. A petri dish contains 100 bacteria, and the number N of bacteria is increasing according to the equation $\frac{dN}{dt} = kN$, where k is a constant and t is measured in hours. At time $t = 3$, there are 181 bacteria. Based on this information, what is the doubling time for the bacteria?

$$N = N_o e^{kt}$$
$$181 = 100e^{3k} \qquad 200 = 100e^{0.1977t}$$
$$1.81 = e^{3k} \qquad 2 = e^{0.1977t}$$
$$\ln(1.81) = 3k \qquad \ln(2) = 0.1977t$$
$$k \approx 0.1977 \qquad \boxed{t \approx 3.506 \text{ hours}}$$

11. A radioactive substance has a rate of decay that can be modeled by the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the half-life of the radioactive substance is 300 years, then what is the value of k?

$$\frac{1}{2} = 1 \cdot e^{k(300)}$$
$$\ln(1/2) = 300k$$
$$\boxed{k \approx -0.0023}$$

12. A dose of 100 milligrams of a drug is administered to a patient. The amount of the drug, in milligrams, in the person's bloodstream at time t, in hours, is given by $A(t)$. The rate at which the drug leaves the bloodstream can be modeled by the differential equation $\frac{dA}{dt} = -0.5A$. Write an expression for $A(t)$.

$$\boxed{A(t) = 100e^{-0.5t}}$$

13. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in five hours, in how many hours will the number of bacteria triple?

$2 = e^{k(5)}$

$\ln(2) = 5k$

$\frac{\ln 2}{5} = k$

$3 = e^{\frac{\ln 2}{5}(t)}$

$\ln 3 = \frac{\ln 2}{5} t$

$\frac{5 \ln 3}{\ln 2} = t$

(A) $\ln\left(\frac{15}{2}\right)$ (B) $\ln\left(\frac{3^5}{2}\right)$ (C) $2\ln(3)\ln 2$ (D) $\frac{\ln 2}{5}$ (E) $\frac{5 \ln 3}{\ln 2}$

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14. A group of tiny organisms (measured in grams) is shrinking a rate modeled by $\frac{dM}{dt} = -0.7M$, where the time t is measured in hours. The size of a second group of organisms decreases at a constant rate of 2 grams per hour according to the linear model $\frac{dN}{dt} = -2$. If at time $t = 0$, the first group has size $M(0) = 4$ and the second colony has size $N(0) = 6$, at what time will both groups of organisms be the same size?

$M(t) = 4e^{-0.7t}$ $N(t) = 6 - 2t$ ← Linear!

Graph and find point of intersection

(A) -0.459 (B) 0.312 (C) 1.292 (D) 2.697