7.9 Logistic Models

Calculus



1. A populations rate of growth is modeled by the logistic differential equation $\frac{dP}{dt} = \frac{1}{400}P(100 - P)$, where t is in days and P(0) = 10. What is the greatest rate of change for this population?

2. Using the logistic differential equation $\frac{dP}{dt} = \frac{1}{3}P - \frac{1}{120}P^2$, what is $\lim_{t \to \infty} P(t)$?

3. A rate of change $\frac{dP}{dt}$ of a population is modeled by a logistic differential equation. If $\lim_{t\to\infty} P(t) = 100$ and the rate of change of the population is 5 when the population size is 20, which of the following differential equations describe the situation?

A.
$$\frac{dP}{dt} = 5P\left(1 - \frac{P}{20}\right)$$

B.
$$\frac{dP}{dt} = 20P \left(1 - \frac{P}{100} \right)$$

C.
$$\frac{dP}{dt} = \frac{5}{16} P \left(1 - \frac{P}{100} \right)$$

D.
$$\frac{dP}{dt} = \frac{16}{5} P \left(1 - \frac{P}{100} \right)$$

4. A rate of change for a population is modeled by the differential equation $\frac{dP}{dt} = 0.3P(66 - P)$. What is the population when the rate of change is the greatest?

5. Which of the following is a logistic differential equation?

$$A. \frac{dP}{dt} = 3t(1-t)$$

$$B. \frac{dP}{dt} = 3P(1-t)$$

$$C. \frac{dP}{dt} = \frac{1}{3}P\left(1 - \frac{t}{50}\right)$$

D.
$$\frac{dP}{dt} = \frac{1}{3}P\left(1 - \frac{P}{50}\right)$$

| Answers to 7 | 7.9 CA #2 |
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| 1. $\frac{25}{4}$ per day 2 | 2. 40 | 3. C | 4. 33 | 5. D |
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