8.11 Washer Method: Revolve Around x or y Axis

Calculus

Solutions

Practice

For each problem, sketch the area bounded by the equations and revolve it around the axis indicated. Find the volume of the solid formed by this revolution. Leave your answers in terms of π.

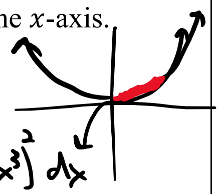
1. $y = x^2,\ y = x^3$. Revolve around the x-axis.

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0 \quad x = 1$



$\pi\int_0^1 (x^2)^2 - (x^3)^2\, dx$

$\pi\int_0^1 (x^4 - x^6)\, dx$

$\pi\left[\frac{x^5}{5} - \frac{x^7}{7}\right]\Big|_0^1$

$\pi\left[\left(\frac{1}{5} - \frac{1}{7}\right) - (0)\right]$

$\pi\left[\frac{7}{35} - \frac{5}{35}\right]$

$\boxed{2\pi/35}$

2. $y = x^2,\ y = x^3$. Revolve around the y-axis.

$x = \pm\sqrt{y} \quad x = \sqrt[3]{y}$

$\pi\int_0^1 (y^{1/3})^2 - (y^{1/2})^2\, dy$

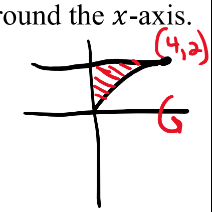
$\pi\int_0^1 (y^{2/3} - y)\, dy$

$\pi\left[y^{\cancel{5/3}} \cdot \frac{3}{5} - \frac{y^2}{2}\right]\Big|_0^1$

$\pi\left[\left(\frac{3}{5} - \frac{1}{2}\right) - (0)\right]$

$\pi\left(\frac{6}{10} - \frac{5}{10}\right) = \boxed{\frac{\pi}{10}}$

3. $y = \sqrt{x},\ x = 0,\ y = 2$. Revolve around the x-axis.



$\pi\int_0^4 (2)^2 - (\sqrt{x})^2\, dx$

$\pi\int_0^4 (4 - x)\, dx$

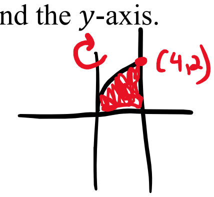
$\pi\left[4x - \frac{x^2}{2}\right]\Big|_0^4$

$\pi\left[(16 - 8) - (0)\right]$

$\boxed{8\pi}$

4. $y = \sqrt{x},\ y = 0,\ x = 4$. Revolve around the y-axis.

$x = y^2$



$\pi\int_0^2 (4)^2 - (y^2)^2\, dy$

$\pi\int_0^2 (16 - y^4)\, dy$

$\pi\left[16y - \frac{y^5}{5}\right]\Big|_0^2$

$\pi\left[\left(32 - \frac{32}{5}\right) - (0)\right]$

$\pi\left(\frac{160}{5} - \frac{32}{5}\right) = \boxed{\frac{128}{5}\pi}$

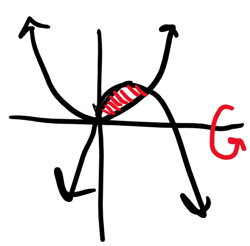
5. $y = x^2$ and $y = 4x - x^2$. Revolve around the x-axis.

$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0 \quad x = 2$



$\pi\int_0^2 (4x - x^2)^2 - (x^2)^2\, dx$

$\pi\int_0^2 (16x^2 - 8x^3 + x^4) - (x^4)\, dx$

$\pi\int_0^2 (16x^2 - 8x^3)\, dx$

$\pi\left[\frac{16}{3}x^3 - 2x^4\right]\Big|_0^2$

$\pi\left[\left(\frac{16}{3}\cdot 8 - 32\right) - (0)\right]$

$\pi\left(\frac{128}{3} - \frac{96}{3}\right)$

$\boxed{\frac{32}{3}\pi}$

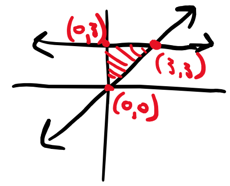
For these problem, sketch the area bounded by the equations and revolve it around the axis indicated. Set up the integral that evaluates the volume of the solid formed by this revolution. Do NOT solve.

6. $y = x,\ x = 0$, and $y = 3$

Revolve around the x-axis.

$\pi \int_0^3 (3)^2 - (x)^2 \, dx$

$\boxed{\pi \int_0^3 (9 - x^2) \, dx}$



Revolve around the y-axis.

$\pi \int_0^3 (y)^2 - 0^2 \, dy$

$\boxed{\pi \int_0^3 y^2 \, dy}$

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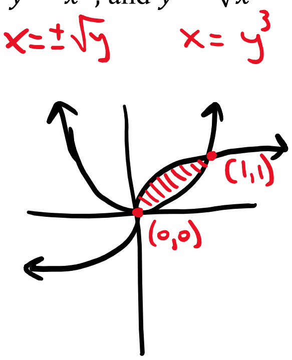
7. $y = x^2$, and $y = \sqrt[3]{x}$

Revolve around the x-axis.

$\pi \int_0^1 (x^{\frac{1}{3}})^2 - (x^2)^2 \, dx$

$\boxed{\pi \int_0^1 (x^{\frac{2}{3}} - x^4) \, dx}$

$x = \pm\sqrt{y}$ $\quad x = y^3$



Revolve around the y-axis.

$\pi \int_0^1 (-\sqrt{y})^2 - (y^3)^2 \, dy$

$\boxed{\pi \int_0^1 (y - y^6) \, dy}$

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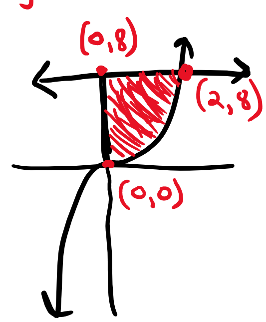
8. $y = x^3,\ x = 0$, and $y = 8$.

Revolve around the x-axis.

$\pi \int_0^2 (8)^2 - (x^3)^2 \, dx$

$\boxed{\pi \int_0^2 (64 - x^6) \, dx}$

$x = y^{\frac{1}{3}}$



Revolve around the y-axis.

$\pi \int_0^8 (y^{\frac{1}{3}})^2 - (0)^2 \, dy$

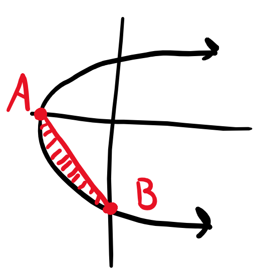
$\boxed{\pi \int_0^8 y^{\frac{2}{3}} \, dy}$

8.11 Washer Method: Revolve Around x or y Axis

Test Prep

$y = \pm\sqrt{x+9}$

9. **Calculator active.** Consider the curve $y^2 = 9 + x$ and the line segment AB joining the points $A(-9,0)$ and $B(0,-3)$ on the curve. The curve and line segment form a bounded region. Find the volume of the solid generated when this region is revolved about the x-axis. [Show the integral set up, and then the answer.]



$\overline{AB} \rightarrow y = -\frac{3}{9}x - 3$

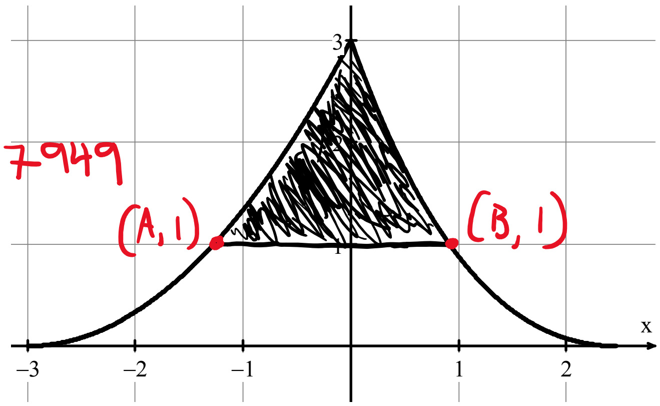
$y = -\frac{1}{3}x - 3$

$\pi \int_{-9}^0 (-\sqrt{x+9})^2 - (-\frac{1}{3}x - 3)^2 \, dx$

$\boxed{42.4115}$

10.

$A \approx -1.267949$



$B \approx 0.9126298$

Calculator active. Let f be the function defined by $f(x) = \begin{cases} \frac{1}{3}(x+3)^2, & -3 \le x < 0 \\ \frac{3}{2}\cos(2\sqrt{x}) + \frac{3}{2}, & 0 \le x < \frac{\pi^2}{4} \end{cases}$

The graph of f is shown above. Let R be the region abounded by the graph of f and the x-axis. The portion of the region R for $1 \le y \le 3$ is revolved about the x-axis to form a solid. Find the volume of the solid, rounded to three decimal places.

$$\pi\int_A^0 \left[\frac{1}{3}(x+3)^2\right]^2 - [1]^2\,dx + \pi\int_0^B \left[\frac{3}{2}\cos(2\sqrt{x})+\frac{3}{2}\right]^2 - (1)^2\,dx$$

$$\boxed{20.164}$$